Stochastic Optimal Control Scheme for Battery Lifetime Extension in Islanded Microgrid via A Novel Modeling Approach

Yuchao Qin, Haochen Hua, *Member, IEEE* and Junwei Cao, *Senior Member, IEEE*

***Abstract*—Most of the existing literatures concerning lifetime extension of battery energy storage (BES) devices mainly focus on designing controllers in BES devices themselves. In this paper, the problem of BES lifetime extension in an islanded microgrid (MG) is considered from a different perspective, in the sense that such target is achieved by adjusting output power of controllable power generation devices. To obtain accurate power models of photovoltaic panels (PVs) and loads, a new hybrid modeling method utilizing both neural networks (NNs) and stochastic differential equations (SDEs) is introduced. We formulate the problem of extending the service life of BES devices as a stochastic optimal control problem. Meanwhile, the situation of over-control in micro-turbines (MTs) is effectively avoided. We solve such stochastic optimization problem using the dynamic programming solver provided by the open source C++ program BOCOPHJB. Finally, the obtained optimal controller is verified with simulations.**

Index Terms*—Neural networks, energy storage, islanded microgrid, optimal control, stochastic systems.

NOMENCLATURE

BES	Battery energy storage.
C_{min}	Lower running constraint for SOC.
C_{max}	Upper running constraint for SOC.
C_{min}^T	Lower terminal constraint for SOC.
C_{max}^T	Upper terminal constraint for SOC.
C_{PLET}^{life}	Total PLET of BES.
DEG	Diesel engine generator.
DOD	Depth of discharge.
FC	Fuel cell.
FES	Flywheel energy storage.
k_P	Peukert lifetime constant.
LOH	Loss of health for BES.
MT	Micro-turbine.
MG	Microgrid.
MLP	Multilayer perceptron.
MSE	Mean square error.
NMSE	Normalized mean square error.
NN	Neural network.
ODE	Ordinary differential equation.
P_{BES}	Input/output power of BES.
P_L	Load power.
P_{Lp}	Deterministic part of P_L.
P_{Le}	Stochastic part of P_L.
PLET	Peukert lifetime energy throughput.
P_{MT}	Output power of MTs.
P_{Max}^{MT}	Maximum power of MTs.
P_{Min}^{MT}	Minimum power of MTs.
P_{in}	Charging power of BES.
P_{out}	Discharging power of BES.
P_{PV}	Output power of PVs.
P_{PVT}	Theoretical solar radiation power.
PV	Photovoltaic panel.
Q_s	Capacity of BES.
r	Ratio of P_{PV} and P_{PVT}.
r_p	Deterministic part of r.
r_e	Stochastic part of r.
R^2	Coefficient of determination.
ReLU	Rectified linear unit.
RES	Renewable energy source.
SDE	Stochastic differential equation.
SOC	State of charge.
T_{MT}	Time constant of MTs.
WTG	Wind turbine generator.
η_{in}	Charging coefficient of BES.
η_{out}	Discharging coefficient of BES.

Manuscript submitted August 8, 2017. This work was supported in part by National Natural Science Foundation of China (grant No. 61472200) and Beijing Municipal Science & Technology Commission (grant No. Z161100000416004).

All of the authors are with Research Institute of Information Technology, Tsinghua University, Beijing 100084, China. (e-mail: jcao@tsinghua.edu.cn).

I. Introduction

With the development of science and technology, human beings have consumed a great deal of fossil fuels in daily life. In recent years, since people are aware of the depletion of the fossil fuels as well as the environmental problems, RESs such as solar and wind power are widely used [1], [2]. Although RESs has advantages such as clean and sustainable, they have defects as well. The power generation by PVs and WTGs mainly depends on weather conditions which are time-varying and sometimes unpredictable. Thus, output power by PVs and WTGs is stochastic and intermittent [3]. In addition, the large reliance on weather conditions leads to weaker controllability of output power from PVs and WTGs, especially for the case of the distributed PVs and WTGs within a certain region.

The concept of MG was proposed to integrate these distributed RESs into the existing power systems, such that they can be fully utilized [4], [5]. Normally, a MG is composed of loads, power generation devices and energy storage devices. MGs can function in the grid-connected mode. Alternatively, when disconnecting with the main grid, MG shall be able to function on its own, which is denoted as the islanded mode, or the off-grid mode [5], [6]. Within one MG, smart monitoring devices such as smart meters are set in loads, power generation devices and energy storage devices, such that the MG system parameters (e.g., power, voltage, etc.) can be obtained. There have been a variety of existing MG demonstration projects, e.g., the MG in Huatacondo, Chile; the MG project in Sendai, Japan; the PV MG system in Hangzhou, China, etc., [7], [8].

The existence of vast RESs makes the dynamical system of MGs more complicated than that of the conventional power grids [3], [8], [9]. Particularly, the scenarios of islanded MGs involve a number of challenging problems worth considering. Up till now, there have been a great number of research outputs focusing on different aspects of islanded MGs. One of the outstanding issues is life time extension for BES devices in an islanded MG.

Within the islanded MG, energy storage devices absorb the power deviation between power generation and consumption, such that power supply-demand balance in MG is achieved [10], [11]. Since most of the energy storage devices are relatively expensive [12], extending the service life of the energy storage devices has been regarded as a significant common target for electrical engineers. Study on the loss of energy storage devices can be found in [10], [13]. Capacity design and optimization of energy storage device have been investigated in [11], [12].

In the field of MGs, the multi-objective optimization problems have been studied in [14], [15]. In terms of system modeling, power consumption of loads is discussed in [16], [17]; and the prediction model regarding the power generation by PVs has been investigated in [18].

The power forecast is another important issue in power systems. Typically, the stochastic nature of the RESs in MGs make it even more remarkable. The technique of NNs has been widely used in power forecasting tasks. In [18], NNs are used to predict the PV output power. Deep learning techniques have been applied to predict the power change of loads [16]. It is notable that the drawback of only using NNs to model the power dynamics (e.g., [16], [18]) is that drastic fluctuations cannot be fully considered.

Apart from the NN techniques, the stochastic process has also been widely employed to formulate the predictive power models. The theory of stochastic analysis was first applied to describe the dynamics of power systems in 1980s [19], [20]. Very recently, the application of SDEs in the field of system modeling for MG has attracted much attention, and significant advances on this topic have been made; readers can consult [7], [21]-[23]. It is notable that only using SDEs to describe the power dynamics of PVs, WTGs and loads is a tough task, since acquiring such complicated SDE is difficult and costly.

In this paper, we consider an islanded MG which is composed of PVs, MTs, BES devices and loads. Our main purpose is to design controllers based on an accurate dynamic system of the considered MG, such that the service life of the BES devices can be extended. Firstly, by applying a hybrid modeling approach, the dynamic system of the considered islanded MG is obtained. Then, the target to extend the service life of batteries is formulated as a stochastic optimal control problem. It is notable that in the considered MG, the controllers are set in MTs only. We solve the stochastic optimization problem via the dynamic programming solver provided by the open source C++ program BOCOPHJB [24]. Finally, the obtained optimal controller is evaluated with numerical simulations.

The importance and contribution of this paper can be highlighted as follows.

1. Most of the related literatures concerning the lifetime extension of BES devices mainly focus on designing controllers in BES devices themselves; see, e.g., [10], [13]. In this paper, the problem of BES lifetime extension is considered from a different perspective by adjusting output power of controllable power generation devices. In the considered islanded MG, this target is achieved by designing control schemes for MTs. In this sense, the existing methods (e.g., [10], [13]) can still be implemented based on our proposed method, such that that the service life of BES devices can be further extended.

2. Some constraints in practical power systems have been taken into account in the studied control problem. The output power of MTs are restricted within its capacity. In order to preserve the reaction ability of BES devices, upper and lower bounds are set for SOC. Under the proposed controller, the situation of over-control in MTs is avoided effectively.

3. It is highlighted that our proposed method can be applied into more complicated MG scenarios (for example, MGs containing multiple WTGs, DEGs, FCs and FES devices) without essential difficulty. This is because the power models of WTGs are similar to that of PVs; power dynamics of controllable devices e.g., DEGs and FCs can be formulated as the linear ODEs similar to power dynamics of MTs; and

the power dynamic equation of FES devices is very similar to that of BES devices; see, e.g., [9], [23].

4. Apart from the main contribution in this paper that successfully extends the service life of BES devices within an islanded MG, we have made some innovative developments on system modelling. Different from most of the conventional articles using either stochastic processes or NNs to describe power dynamics, in this paper, the technique of utilizing both SDEs and NNs *simultaneously* has been proposed to establish an accurate power model for PVs and loads. Compared with the existing works that deal with the modeling of MGs using SDEs, e.g., [7], [21], our approach additionally introduces the MLP to model the power of PVs and loads. Compared with some existing works using NNs to model the power of PVs and loads, e.g., [16], [18], our approach additionally introduces SDE to include the drastic fluctuations.

The rest of the paper is organized as follows. Section II describes the modeling for the dynamics of the islanded MG. Section III formulates the stochastic optimal control problem and introduces the approach to solving it numerically. Section IV provides some numerical simulations and summarizes the outcomes. Finally, the conclusion is presented in Section V.

II. System Modeling of Islanded MG

In this section, we formulate the power models of PVs, loads, MTs and BES devices, by which we obtain the mathematical model of the considered islanded MG.

A. The Components of the Islanded MG and the Source of Data

The islanded MG considered in this paper is composed of distributed PVs, loads, MTs and BES devices. The controllers are assumed to be set in MTs only. Both PVs and BES devices are considered to be uncontrollable. Since the response time of batteries is much faster than that of MTs [25], [26], the drastic power fluctuations in islanded MG is absorbed by BES devices, rather than eliminated by MTs. The output power of MTs can be controlled to achieve a long-term power supply-demand balance in the islanded MG.

The data used in this paper come from the database provided by Pecan Street Inc. [27]. We focus on totally 280 civilian buildings that joint the smart grid project in Austin, Texas, US. The daily power data of these buildings (PVs, loads) and the local weather data from January 1st to December 31st in 2016 are obtained from [27]. The unit of PV power generation and power usage is kW and the power data are sampled at a frequency of 1/60 Hz. The weather data include weather summary, temperature, humidity, atmosphere pressure, wind speed, cloud cover and probability of precipitation, all of which are recorded once an hour. The theoretical solar irradiation is confirmed to be effective for PV power modeling [18]. Since the real solar irradiation data are not provided in the data source [27], the theoretical solar irradiation data calculated based on the geographic location and local time are used for the PV output power modeling.

In the following subsections, the power models of PVs and loads are formulated based on real data [27], whereas the power models of MTs and BES devices are obtained according to the existing literatures [3] and [7], respectively.

B. Power Modeling of PVs

The PV output power curves normally contain drastic fluctuation, and the size of fluctuation varies during the day time. Since the output power of PVs is directly related with the power of solar irradiation, PV output power during night time is zero, leading to the failure of only using SDEs to model PV power for the whole day. Due to this feature, we propose a new method to describe PV output power.

In order to describe such characteristics of PV output power, we propose the following model,

$$P_{PV}(t) = P_{PVT}(t)r(t), \tag{1}$$

where $P_{PV}(t)$ stands for the output power of PVs at time t, and $P_{PVT}(t)$ represents the theoretical solar irradiation power received by PVs at time t ignoring weather conditions. In (1), $r(t)$ is defined to be the ratio of $P_{PV}(t)$ and $P_{PVT}(t)$. Mathematically, in order to obtain the power model of PVs, we have to find out the varying pattern of $r(t)$.

We divide the varying pattern of $r(t)$ into two parts. The first part is $r_p(t)$, which represents for the overall trend. The second part is $r_e(t)$, which is the stochastic deviation. Mathematically, we have the following relationship,

$$r(t) = r_p(t) + r_e(t). \tag{2}$$

Next, we focus on finding the models of $r_p(t)$ and $r_e(t)$, respectively.

Firstly, MLP is applied to obtain the model of $r_p(t)$. The detailed steps are given as follows. According to the working mechanism of PVs, weather conditions like temperature, cloud cover, etc., shall influence the PV output power, leading to the change of $r_p(t)$. We assume that there exists a functional relationship between some of the above weather conditions and $r_p(t)$. MLP is used to match such functional relationship.

To find the weather data that are relevant with $r_p(t)$, we adopt the similar approach introduced in [18]. The smoothed $r(t)$ series (denoted as $r_p^*(t)$) are used as the approximated values of $r_p(t)$. We calculate the Pearson product-moment correlation coefficients (see Appendix A) between $r_p(t)$ and the weather data. The results are shown in Table I.

According to Table I, we find that temperature, humidity, cloud cover and probability of precipitation have relatively strong relationship with $r_p(t)$. Thus, we use the above four types of weather records as the input of the MLP and use the output of the network as the estimation of $r_p(t)$. The MLP contains one input layer, one output layer and two hidden layers. The ReLU function is chosen as the activation function for neurons in the MLP network. We train such NNs with backpropagation (BP) algorithm such that the MSE between the outputs of the MLP and the target values $r_p^*(t)$ is minimized. Then, we are able to regard the output of the NNs as $r_p(t)$. In this sense, the model of $r_p(t)$ is obtained.

TABLE I
THE CORRELATION COEFFICIENTS.

Weather Information	Correlation Coefficients
Temperature	0.323
Humidity	−0.599
Atmosphere Pressure	0.029
Wind Speed	−0.038
Cloud Cover	−0.617
Probability of Precipitation	−0.350

The coefficient of determination R^2 (see Appendix B) is used as the criteria to evaluate the prediction accuracy of the MLP model. We randomly select 20% of all valid data as the test set and the remaining as the training set. We perform the training and validation of the MLP on the training set and the test set, respectively. The R^2 values for the trained MLP model are shown in Table II.

TABLE II
THE R^2 VALUE OF THE ESTIMATION FOR $r_p(t)$.

Data Set	R^2
Training Set	0.718
Test Set	0.712

Next, we focus on obtaining the model of $r_e(t)$. We choose the continuous time SDE driven by Brownian motion to describe such stochasticity as follows,

$$\mathrm{d}r_e(t) = \mu_1 r_e(t)\mathrm{d}t + \sigma_1 \mathrm{d}w_1(t), \tag{3}$$

where $w_1(t)$ is a standard Brownian motion, μ_1 and σ_1 are system parameters to be determined.

Taking a small time step Δt, we obtain a discrete version of SDE (3) as follows,

$$\Delta r_e(t) = \mu_1 r(t)\Delta t + \sigma_1\big(w_1(t+\Delta t) - w_1(t)\big). \tag{4}$$

The term $\Delta r_e(t)$ in (4) represents the increment of $r_e(t)$ at time t. According to the properties of Brownian motion, the increment $w_1(t+\Delta t) - w_1(t)$ follows the normal distribution $\mathcal{N}(0, \Delta t)$. In this sense, we are able to calculate the transition probability $\mathcal{P}\big(r_e(t+\Delta t)|r_e(t)\big)$.

We estimate the values of μ_1 and σ_1 with the maximum likelihood estimation method. Suppose that there is a sample sequence set of $r_e(t)$, which we denote as $\{r_e^0, r_e^1, \dots, r_e^N\}$. Given μ_1 and σ_1, the log-likelihood function is given in (5).

$$\ln\big(L(\mu_1,\sigma_1)\big) = \sum_{t=1}^{N} \ln\big(\mathcal{P}(r_e^t | r_e^{t-1}, \mu_1, \sigma_1)\big). \tag{5}$$

Next, we are about to find a pair of μ_1 and σ_1 that maximize $\ln\big(L(\mu_1,\sigma_1)\big)$. By assigning the two partial differential derivatives $\frac{\partial \ln(L(\mu_1,\sigma_1))}{\partial \mu_1}$ and $\frac{\partial \ln(L(\mu_1,\sigma_1))}{\partial \sigma_1}$ to zero, we can find the solutions in (6).

$$\begin{cases} \mu_1 = \dfrac{\sum_{t=1}^{N}(r_e^t - r_e^{t-1})r_e^{t-1}}{\sum_{t=1}^{N} r_e^{t-1}\Delta t}, \\ \sigma_1 = \sqrt{\dfrac{\sum_{t=1}^{N}\left(r_e^t - r_e^{t-1} - \mu_1 r_e^{t-1}\Delta t\right)^2}{\Delta t}}. \end{cases} \tag{6}$$

Generally, under different weather conditions, for example, sunny or cloudy, the power generation of PVs would have some different characteristics. According to the records in [27], the weather conditions are roughly divided into four categories: rainy, sunny, partly cloudy and mostly cloudy. In order to achieve a better approximation for the dynamics of the PV power generation, we associate the power data with these different weather conditions and calculate the system parameters under different weather types. Parameters for (3) under different weathers are provided in Appendix C.

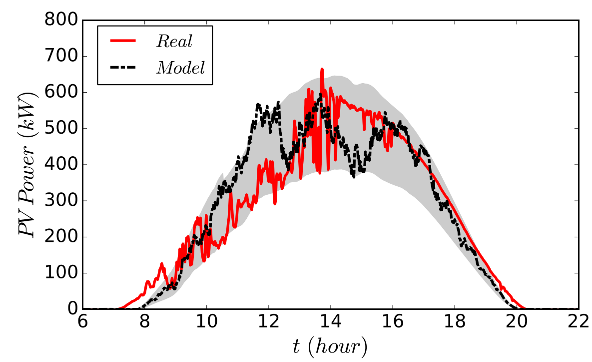


Fig. 1. Typical simulated curves of the proposed PV power model.

In Fig. 1, we plot one typical simulated power curve for our proposed PV power model together with the real PV power curve within the same day. In Fig. 1, the notation *Real* stands for the raw data of PV power in [27], and the notation *Model* refers to one of the possible PV power curves obtained with our proposed method. The grey shadowed area in Fig. 1 stands for the most possible range of our proposed PV power model, mathematically referring to $P_{PVT}(t)r_e(t)$. Obviously, the simulated examples are very close to the real data, which indicates the feasibility and reliability of the proposed PV power model.

C. Power Modeling of Loads

The variation of load power is closely related to the behavior of the inhabitants in each building. The random turning on and off of a large number of electrical devices will lead to load power deviations. So, it is difficult to use deterministic systems to model the load power. Similar to the modeling of PV power, we suggest the following model,

$$P_L(t) = P_{Lp}(t) + P_{Le}(t), \tag{7}$$

where $P_L(t)$ is the desired power model of loads, $P_{Lp}(t)$ is the overall trend of load power, and $P_{Le}(t)$ is the stochastic deviation of load power.

Next, we focus on finding the models of $P_{Lp}(t)$ and $P_{Le}(t)$, respectively. To avoid duplicated expression, we briefly introduce the following steps, since the main idea is similar to that of the PV power modeling.

Firstly, we apply the MLP to obtain the model of $P_{Lp}(t)$. The structure and parameters of the MLP in this section are similar with the ones in the PV modeling. The MLP contains two hidden layers and uses ReLU as the activation function. We denote the smoothed load power data as $P^*_{Lp}(t)$ and treat $P^*_{Lp}(t)$ as the fitting target of the network output.

Similar with the modeling of PV power, we choose some appropriate weather information as the inputs of the NNs. Then, we train such NNs with BP algorithm, such that the MSE between the outputs and $P^*_{Lp}(t)$ is minimized. Then, the outputs of the NNs are denoted as $P_{Lp}(t)$. To show the accuracy of the trained model, the R^2 values for the estimation of $P_{LP}(t)$ are shown in Table III.

TABLE III
THE R^2 VALUE OF THE ESTIMATION FOR $P_{LP}(t)$.

Data Set	R^2
Training Set	0.877
Test Set	0.876

Next, we focus on obtaining the model of $P_{Le}(t)$. We choose the continuous SDE to describe such stochasticity. We have

$$\mathrm{d}P_{Le}(t) = \mu_2 P_{Le}(t)\mathrm{d}t + \sigma_2 \mathrm{d}w_2(t). \tag{8}$$

The remaining approaches are similar to that of modeling $r_e(t)$ for PV power from (4) to (6). Here, we omit the details. We combine the model of $P_{Lp}(t)$ with that of $P_{Le}(t)$, then the power modeling of $P_L(t)$ is obtained. Based on the proposed load power model, we plot the estimation results of load power in Fig. 2.

In Fig. 2, the notation $Real$ stands for the real data of load power in [27], and the notation $Model$ refers to one of the possible load power curves obtained using our proposed model. The grey shadowed area in Fig. 2 stands for the most possible range of our proposed load model, mathematically referring to $P_{Le}(t)$.

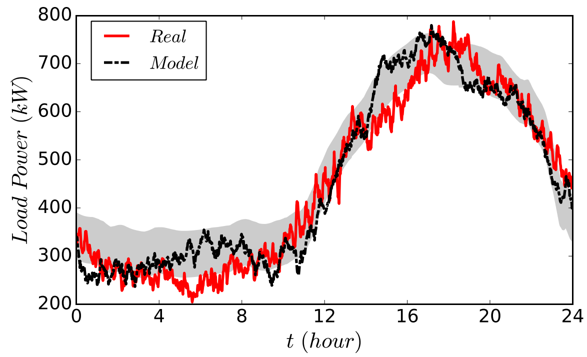


Fig. 2. Typical simulated power curves of load.

Obviously, the simulated examples are very close to the real data, which indicates the feasibility and reliability of the proposed load power model.

D. The Power Modeling of MTs

The MTs are able to provide stable electricity to the islanded MG, thus the output power of MTs is controllable [3]. In our considered islanded MG, when PVs cannot provide energy during the night time, MTs play a significant role in power supply side.

In this paper, our purpose is to extend the service life of BES devices, and the core to achieve this is to make sure that the power deviation in BES devices is controlled within a certain range. Such requirement pushes us to design a controller in the operation system of the islanded MG. Since it is difficult to control the output power of the PVs directly [3], we assume that such controller is set in MTs only. The importance of setting controllers in MTs is that conventional control policies within the BES devices (e.g., [10], [13]) can still be implemented based on our proposed method.

Similar to the power modeling of MTs introduced in [3], we adopt the following linear ODE to describe the power model for MTs,

$$\mathrm{d}P_{MT}(t) = -\frac{1}{T_{MT}}\left(P_{MT}(t) - P_{MT}^{Max}u(t)\right)\mathrm{d}t, \tag{9}$$

where $P_{MT}(t)$ is denoted as output power of MTs, whose value belongs to $[0, P_{MT}^{\mathrm{Max}}]$. The time-varying control input signal is denoted as $u(t)$.

E. The Modeling of BES Devices.

In general, the full capacity of BES devices will be gradually reduced during the whole battery lifetime [13]. Since our attention is only paid to the short-term dynamics of BES devices, the capacity loss of batteries is not taken into consideration in this paper.

The SOC at time t is denoted as $SOC(t)$, and $SOC(t) \in [0,1]$. The power flow into (or, out of) the BES devices at time t is denoted as $P_{BES}(t)$. The charging and discharging power of BES devices can be obtained from (10) and (11), respectively.

$$P_{in}(t) = \max\left(0, P_{BES}(t)\right), \tag{10}$$

$$P_{out}(t) = \max\big(0, -P_{BES}(t)\big). \quad (11)$$

Similar to the modeling of energy storage devices in [28], we adopt the following ODE to describe the power model of BES devices,

$$\mathrm{d}SOC(t) = \frac{\eta(P_{BES}(t))P_{BES}(t)}{Q_S}\mathrm{d}t. \quad (12)$$

The coefficient $\eta\big(P_{BES}(t)\big)$ in (12) is defined as follows,

$$\eta\big(P_{BES}(t)\big) \triangleq \begin{cases} \eta_{in} \ , & P_{BES}(t) \geq 0, \\ 1/\eta_{out}, & P_{BES}(t) < 0. \end{cases} \quad (13)$$

The quantification criteria for the lifetime of BES devices is introduced in Section III-C.

III. THE STOCHASTIC OPTIMAL CONTROL PROBLEM FOR ISLANDED MG

In this section, we formulate the integrated islanded MG control system and the objective function mathematically. In this sense, the issue of extending the service life of BES devices is transformed into a stochastic optimal control problem which can be solved by the dynamic programming principle.

A. The System Modeling of Islanded MG

Based on the power modeling of PVs, loads, MTs, and BES devices, we suggest the following system for our considered islanded MG. Let $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_t)$ be a given complete filtered probability space, where there exist two scalar Brownian motions $w_1(t)$ and $w_2(t)$, $0 \leq t \leq T$. We assume that $w_1(t)$ and $w_2(t)$ are independent. Consider the following equations:

$$\begin{cases} P_{PV}(t) = P_{PVT}(t)\Big(r_p(t) + r_e(t)\Big), \\ P_L(t) = P_{Lp}(t) + P_{Le}(t), \\ \mathrm{d}r_e(t) = \mu_1 r_e(t)\mathrm{d}t + \sigma_1 \mathrm{d}w_1(t), \\ \mathrm{d}P_{Le}(t) = \mu_2 P_{Le}(t)\mathrm{d}t + \sigma_2 \mathrm{d}w_2(t), \\ \mathrm{d}P_{MT}(t) = -\frac{1}{T_{MT}}\big(P_{MT}(t) - P_{MT}^{Max}u(t)\big)dt, \\ \mathrm{d}SOC(t) = \frac{\eta(P_{BES}(t))P_{BES}(t)}{Q_S}\mathrm{d}t. \end{cases} \quad (14)$$

In general, the dynamic response time of BES devices is less than one second [9], [25]. Noticing that the proposed model describes the dynamics of the MG system at minute level, the power deviation could be absorbed by the BES devices almost instantly. Thus, the balance between power generation and consumption can be formulated by

$$P_{MT}(t) + P_{PV}(t) - P_L(t) - P_{BES}(t) = 0. \quad (15)$$

Mathematically, (14) and (15) can be transformed into the following control system (16). We define an admissible controller $u(\cdot)$ to be any $\mathcal{F}_t$-adapted process under which (16) has a unique solution. The set of all admissible controls is denoted by $\mathcal{U}$. We have

$$\begin{cases} \mathrm{d}x(t) = (Ax(t) + Bu(\cdot) + C)\mathrm{d}t + D\mathrm{d}W(t), \\ x(0) = x_0, \end{cases} \quad (16)$$

where $x(t) = [P_{Le}(t) \quad r_e(t) \quad P_{MT}(t) \quad SOC(t)]'$ is system state, $u(\cdot)$ is the control input. In the diffusion part of (16), the Brownian motion $W(t) = [w_1(t) \quad w_2(t)]'$.

System coefficients A, B, C, D in (16) are of the following forms:

$$A = \begin{bmatrix} \mu_2 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & -1/T_{MT} & 0 \\ -a(x(t),t) & a(x(t),t)P_{PVT}(t) & a(x(t),t) & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{P_{MT}^{Max}}{T_{MT}} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ c(x(t),t) \end{bmatrix}, \quad D = \begin{bmatrix} \sigma_2 & 0 \\ 0 & \sigma_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Besides, we have

$$a(x(t),t) = \frac{\eta\big(P_{BES}(t)\big)}{Q_S},$$

and

$$c(x(t),t) = a(x(t),t)\Big(r_p(t)P_{PVT}(t) - P_{Lp}(t)\Big).$$

B. Constraints for the Islanded MG Model

Practically, SOC should be maintained within a proper range, such that power balance can be ensured in the islanded MG and the overcharge/over-discharge of BES devices can be avoided. We set the running constraints for the BES devices as $C_{Min} \leq SOC(t) \leq C_{Max}$. In order that the BES is of adequate ability to deal with the power oscillation, the terminal constrains for SOC is set to be within a strict range $C_{Min}^T \leq SOC(t) \leq C_{Max}^T$.

On the other hand, the islanded MG system might become uncontrollable when all MTs are shut down. It costs a certain amount of fuel to turn MTs on as well. Besides, a certain amount of time is required for the MTs to restart [28]. To prevent these challenges, the minimum output power of MTs is set to be P_{MT}^{Min}.

The constraints for the MG system are given in (17)

$$e_1 \leq E_1 x(t) \leq e_2, \quad e_3 \leq E_2 x(T) \leq e_4, \quad (17)$$

where

$$E_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_2 = [0 \quad 0 \quad 0 \quad 1],$$

$$e_1 = \begin{bmatrix} P_{MT}^{Min} \\ C_{Min} \end{bmatrix}, \quad e_2 = \begin{bmatrix} P_{MT}^{Max} \\ C_{Max} \end{bmatrix},$$

$$e_3 = C_{Min}^T, \quad e_4 = C_{Max}^T.$$

C. The Formulation of Cost Function

In general, the service life of BES devices refers to the battery cycle count during its normal operation. Since the total battery cycle count is limited and the BES devices are expensive [12], it is important to develop an optimal control scheme within MTs such that the BES devices can be utilized rationally. Mathematically, as long as the expected value for the lifetime loss of BES devices is minimized, we claim that the so-called BES rational utilization is achieved.

A variety of criteria concerning the lifetime of BES devices have been discussed in existing literatures. For example, in [13], the charging/discharging power of BES devices as well as the deviation of SOC are taken into account. In [10], the Peukert lifetime energy throughput (PLET) model is used to measure the decrement of BES lifetime. In this paper, we employ the PLET model introduced in [10] as the criterion to evaluate the loss of BES devices.

In the PLET model, the BES lifetime is relevant with C_{PLET}^{life} which is defined in (18),

$$C_{PLET}^{life} \triangleq nd^{k_P}, \tag{18}$$

where d is DOD of BES devices ($DOD = 1 - SOC$); n is the total number of charging/discharging cycles during the lifetime of BES devices [10]. According to [10], k_P is typically in range of 1.1 to 1.3, and the total energy throughput C_{PLET}^{life} is almost constant for any specific d. Thus, it can be used to measure the LOH for BES devices [10].

Consider the ith discharge process of BES devices during the time period $t \in [0, T]$. The DOD of this process is assumed to be $d_i = \sum_j \Delta d_{i,j}$. According to [10], the following formula is established,

$$\operatorname{argmin}\left(d_i^{k_P}\right) \approx \operatorname{argmin}\left(\sum_j \left(\Delta d_{i,j}\right)^{k_P}\right). \tag{19}$$

For the PLET increment Δc_{PLET} in the considered time period can be further approximated with (20).

$$\Delta c_{PLET} \approx \sum_i \sum_j \left(\Delta d_{i,j}\right)^{k_P} \approx \int_0^T \left(\frac{P_{out}(t)}{\eta_{out} Q_S}\right)^{k_P} \mathrm{d}t\,. \tag{20}$$

The LOH within a considered period can be calculated with

$$LOH(\%) = \frac{\Delta c_{PLET}}{C_{PLET}^{life}} \times 100\%. \tag{21}$$

Large growth of LOH implies fast decrease of BES lifetime. In order to extend the service life of BES devices, LOH should be minimized. Meanwhile, instead of setting controllers in BES devices (e.g., [10], [13]), in this paper, the control target is considered within the scope of the whole MG, and the controller is set in MTs. Thus, the control cost for MTs shall be considered while minimizing the LOH of BES devices.

To achieve the trade-off between the lifetime extension of BES and the rational utilization of MTs, the objective cost function $J\big(0, x(0); u(\cdot)\big)$ is defined as follows, (time t omitted)

$$J\big(0, x(0); u(\cdot)\big) \triangleq \mathbb{E}\left[\int_0^T [\left(\frac{P_{out}(t)}{\eta_{out} Q_S}\right)^{k_P} + \alpha u^2]\, \mathrm{d}t\right], \tag{22}$$

where $\mathbb{E}$ stands for the mathematical expectation, and α is a weighting coefficient for the controller in MTs. It is notable that system state $x(t)$ is a stochastic process. Thus, the associated cost function must be measured by its expected value. The first integrant $\left(\frac{P_{out}(t)}{\eta_{out} Q_S}\right)^{k_P}$ in (22) is used to calculate the LOH of BES at each time step. The second integrant αu^2 in (22) is used to restrict the strength of the controller such that the situation of over-control is avoided.

Our target is to find an optimal control policy for MTs such that the cost function $J\big(0, x(0); u(\cdot)\big)$ is minimized.

D. Solving the Stochastic Optimal Control Problem

Now we have transformed our practical scenario into a stochastic optimal control problem. Our aim is to minimize the cost function $J\big(0, x(0); u(\cdot)\big)$ subject to (s.t.) (16) and (17), i.e.,

$$\begin{aligned} \min_{u(\cdot)\in\mathcal{U}} \quad & J\big(0, x(0); u(\cdot)\big), \\ \text{s.t.} \quad & \mathrm{d}x(t) = \big(Ax(t) + Bu(\cdot) + C\big)\mathrm{d}t + D\mathrm{d}W(t), \\ & e_1 \le E_1 x(t) \le e_2, \\ & e_3 \le E_2 x(T) \le e_4, \end{aligned} \tag{23}$$

where $x(0) = x_0$.

An admissible pair $\big(x^*(\cdot), u^*(\cdot)\big)$ is called optimal (w.r.t. the initial condition $(0, x_0)$ if $u^*(\cdot)$ achieves the infimum of $J\big(t, x(t); u(\cdot)\big)$ in (23).

Using the BOCOPHJB toolbox [24], through discretization and dynamic programming techniques, the proposed optimal control problem (23) is solved.

Remark: The disadvantage of the BOCOPHJB toolbox is that the complexity of computation and storage space might increase exponentially with the increase of the system's dimension. In this paper, the system that we studied is 4-dimensional, which is acceptable for the application of the BOCOPHJB toolbox.

IV. Numerical Simulations

In this section, numerical simulations are provided to show the feasibility and effectiveness of the proposed method.

The time range for the simulation is set to be [8,18] (hours) and the time step is set to be 30s. The MG model obtained in Section II is used to generate the power dynamics of PVs and loads for simulations. The initial value of SOC is assigned to be 0.6. Data on August 21st 2016 in [27] are chosen for simulation. According to the records, the weather during that period belongs to the rainy case. The basic system parameters for simulation are shown in Table IV.

TABLE IV
SYSTEM PARAMETERS

Parameters	Values	Parameters	Values
T_{MT}	$3\ min$	C_{Min}	0.3
P^{MT}_{Max}	$600\ kW$	C_{Max}	0.8
P^{MT}_{Min}	$6\ kW$	C^{T}_{Min}	0.5
C^{life}_{PLET}	605.23	C^{T}_{Max}	0.8
k_P	1.15	η_{in}	0.97
α	0.1	η_{out}	0.95
Q_S	$500\ kWh$		

Additionally, to show the advantages of the proposed stochastic system model over the conventional deterministic one, the simulation results under the corresponding deterministic controller is provided. To obtain the deterministic controller, the deterministic system *degenerated* from (23) is taken into consideration. Here, *degenerated* means that the volatility term $DdW(t)$ in (23) is eliminated. Thus, the stochastic characteristics of the islanded MG system are not considered in the degenerated system.

Both of the deterministic and stochastic control schemes are obtained with the solver provided in [24]. To demonstrate the advantages of our proposed method, the performances of the control schemes for the stochastic system and the degenerated deterministic system are evaluated in same environment.

A. Simulation under the Proposed Control Scheme

The power dynamics of PVs, MTs and loads under the proposed control scheme are illustrated in Fig. 3. The SOC curve under the proposed control law is illustrated in Fig. 4.

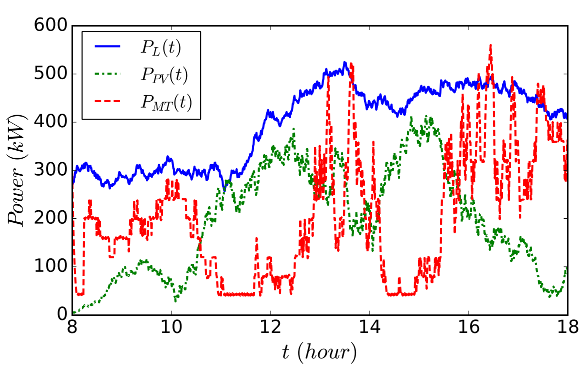


Fig. 3. Power dynamics of PVs, MTs and loads under the proposed control scheme.

The value of the cost function in this simulation is 66.798. The LOH in this simulation is 0.02689%. The average LOH in 200 simulations is 0.02875%.

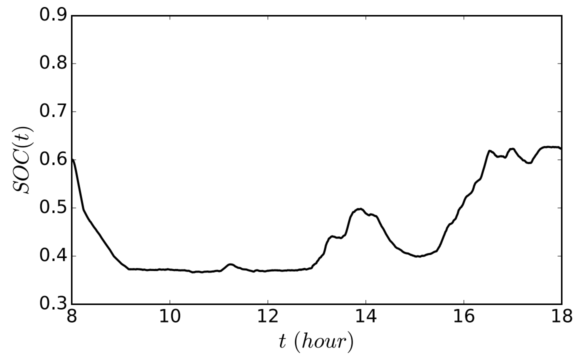


Fig. 4. The SOC curve of BES devices under the proposed control scheme.

B. Simulation under the Deterministic Control Scheme

The SOC curve under the optimal control law is illustrated in Fig. 5.

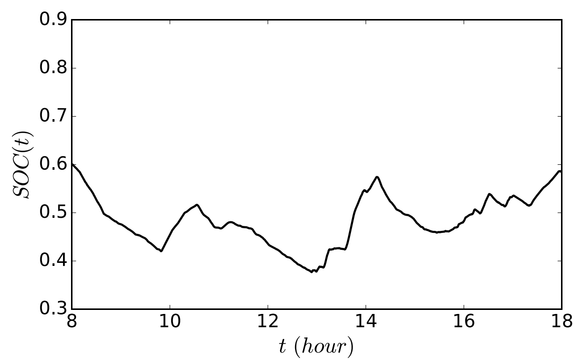


Fig. 5. The SOC curve of BES devices under deterministic control scheme.

The value of the cost function in this simulation is 75.100. The LOH in this simulation is 0.03025%. The average LOH in 200 rounds of simulation is 0.03694%.

The dynamics of SOC in Fig. 4 and Fig. 5 show that under the proposed control scheme, the cumulative energy throughput of BES devices is smaller than that via the deterministic control scheme. In this sense, the lifetime of the BES devices can be further prolonged via the proposed control scheme. It is clear that the value of cost function for the stochastic control scheme is smaller. Besides, both of the single and average LOH under the proposed scheme is smaller than that under the deterministic one.

With the above simulation results, the advantages of the proposed stochastic control method for the lifetime extension of BES devices are demonstrated. Although the general trends of the PV power and load power are taken into account in the deterministic control scheme, the deterministic controller does not impair the losses of BES devices conspicuously. Oppositely, our proposed control method effectively reduces the LOH for BES devices in the simulations.

According to the comparison and analysis of the above simulated results, the proposed control method for the investigated stochastic optimal control problem appears to be effective. The reasonable utilization of BES devices is achieved along with the normal operation of the islanded MG.

V. Conclusions

In summary, we transform the practical islanded MG system into a class of stochastic control systems. We formulate a stochastic optimization problem such that the LOH for the BES in the studied MG is minimized. Meanwhile, the SOC is controlled within a reasonable range. It is highlighted that a new hybrid method modeling the power of PVs and loads is proposed using both NNs and SDEs simultaneously. By using the proposed control scheme, the service life of BES devices can be effectively extended. The numerical simulations demonstrate the usefulness of our proposed approach.

In order to apply the proposed approach to more complicated scenarios, designing more accurate and efficient algorithms for the solution of the proposed stochastic optimal control problem will be the main direction for our future work.

Appendix

A. Pearson Product-Moment Correlation Coefficient

Assuming that there are totally N pairs of data X and Y, The Pearson product-moment correlation coefficient between X and Y are calculated by

$$R = \frac{\sum_{i=1}^{N}(X_i - \bar{X}) \times (Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \bar{X})^2} \times \sqrt{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}},$$

where R is the Pearson product-moment correlation coefficient; $\bar{X}$ and $\bar{Y}$ stand for the average values of X and Y, respectively.

B. Coefficient of Determination

Assuming that there are a series of data $\{y(t_k), k = 1,2,\ldots,N\}$ and the estimated value $\{\hat{y}(t_k), k = 1,2,\ldots,N\}$ which is obtained with a predictive model M. The coefficient of determination R^2 for these estimations from model M is calculated with the following formula

$$R^2 = 1 - \frac{\sum_{k=1}^{N}\left(\hat{y}(t_k) - y(t_k)\right)^2}{\sum_{k=1}^{N}(\mathrm{y}(t_k) - \bar{y})^2},$$

where $\bar{y}$ is the mean value of the data series $\{y(t_k), k = 1,2,\ldots,N\}$.

C. Parameters for Power Models of PVs and Loads

TABLE V
PARAMETERS FOR POWER MODELS OF PVS AND LOADS IN FOUR WEATHER TYPES.

Parameters	Weather Types			
	Rainy	Sunny	Partly cloudy	Mostly cloudy
μ_1	-2.459	-3.278	-1.987	-4.896
σ_1	0.151	0.095	0.148	0.189
μ_2	-0.520	-0.726	-0.515	-0.579
σ_2	44.80	38.16	40.38	47.44

Acknowledgement

This work was supported in part by National Natural Science Foundation of China (grant No. 61472200) and Beijing Municipal Science & Technology Commission (grant No. Z161100000416004).

References

[1] K. Kaygusuz, “Renewable energy: Power for a sustainable future,” *Energy Exploration & Exploitation*, vol. 19, no. 6, pp. 603-626, 2001.

[2] R. Cardenas, R. Peña, S. Alepuz, and G. Asher, “Overview of control systems for the operation of DFIGs in wind energy applications,” *IEEE Trans. Ind. Electron.*, vol. 60, no. 7, pp. 2776-2798, Nov. 2013.

[3] H. Bevrani, M. R. Feizi, and S. Ataee, “Robust frequency control in an islanded microgrid: H_∞ and μ-synthesis approaches,” *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 706-717, Mar. 2016.

[4] R. H. Lasseter and P. Paigi, “Microgrid: A conceptual solution,” in *Proc. IEEE 35th Annu. Power Electron. Specialists Conf.*, Aachen, Germany, 2004, pp. 4285-4290.

[5] J. A. P. Lopes, C. L. Moreira, and A. G. Madureira, “Defining control strategies for microgrids islanded operation,” *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 916-924, May 2006.

[6] N. Eghtedarpour and E. Farjah, “Distributed charge/discharge control of energy storages in a renewable-energy-based DC micro-grid,” *IET Renew. Power Gen.*, vol. 8, no. 1, pp. 45-57, Jan. 2014.

[7] B. Heymann, J. F. Bonnans, F. Silva, and G. Jimenez, “A stochastic continuous time model for microgrid energy management,” in *Proc. 2016 European Control Conf.*, Aalborg, Denmark, 2016.

[8] M. Barnes, *et al.*, “Real-world microgrids-an overview,” in *Proc. IEEE Int. Conf. on System of Systems Engineering*, San Antonio, TX, USA, 2007.

[9] H. Bevrani, and T. Ise, *Microgrid Dynamics and Control*, John Wiley & Sons, Hoboken, NJ, USA, 2017.

[10] D. Tran and A. M. Khambadkone, “Energy management for lifetime extension of energy storage system in micro-grid applications,” *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1289-1296, Sept. 2013.

[11] B. Bahmani-Firouzi and R. Azizipanah-Abarghooee, “Optimal sizing of battery energy storage for micro-grid operation management using a new improved bat algorithm,” *Int J. Electr. Power Energy Syst.*, vol. 56, pp. 42-54, 2014.

[12] B. Zhao, X. Zhang, J. Chen, C. Wang, and L. Guo, “Operation optimization of standalone microgrids considering lifetime characteristics of battery energy storage system,” *IEEE Trans. Sustain. Energy*, vol. 4, no. 4, pp. 934-943, Oct. 2013.

[13] Q. Wei, D. Liu, Y. Liu, and R. Song, “Optimal constrained self-learning battery sequential management in microgrid via adaptive dynamic programming,” *IEEE/CAA J. Autom. Sinica*, vol. 4, no. 2, pp. 168-176, Apr. 2017.

[14] A. Chaouachi, R. M. Kamel, R. Andoulsi, and K. Nagasaka, “Multiobjective intelligent energy management for a microgrid,” *IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1688-1699, Apr. 2013.

[15] A. A. Moghaddam, A. Seifi, T. Niknam, and M. R. A. Pahlavani, “Multi-objective operation management of a renewable MG (micro-grid) with back-up micro-turbine/fuel cell/battery hybrid power source,” *Energy*, vol. 36, no. 11, pp. 6490-6507, 2011.

[16] D. L. Marino, K. Amarasinghe, and M. Manic, “Building energy load forecasting using deep neural networks,” in *Proc. 42nd Annu. Conf. of IEEE Ind. Electron. Soc.*, Florence, Italy, 2016.

[17] M. Zhai, “A new method for short-term load forecasting based on fractal interpretation and wavelet analysis,” *Int J. Electr. Power Energy Syst.*, vol. 69, pp. 241-245, 2015.

[18] H. Zhu, *et al.*, “A power prediction method for photovoltaic power plant based on wavelet decomposition and artificial neural networks,” *Energies*, vol. 9, no. 1, p. 11, 2015.

[19] R. Billinton and P. R. S. Kuruganty, "Probabilistic assessment of transient stability in a practical multimachine system," *IEEE Trans. Power App. Syst.*, no. 7, pp. 3634-3641, Jul. 1981.

[20] F. Wu and Y. Tsai, "Probabilistic dynamic security assessment of power systems-I: Basic model," *IEEE Trans. Circuits Syst.*, vol. 30, no. 3, pp. 148-159, Mar. 1983.

[21] T. Odun-Ayo and M. L. Crow, "Structure-preserved power system transient stability using stochastic energy functions," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1450-1458, Aug. 2012.

[22] F. Milano and R. Zárate-Miñano, "A systematic method to model power systems as stochastic differential algebraic equations," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4537-4544, Jun. 2013.

[23] H. Hua, J. Cao, G. Yang, and G. Ren, "Voltage control for uncertain stochastic nonlinear system with application to energy Internet: Nonfragile robust H_∞ approach," *J. Math. Anal. Appl.*, vol. 463, no. 1, pp. 93-110, 2018.

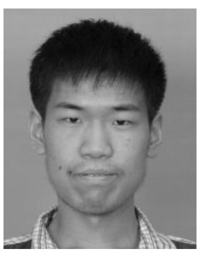
[24] F. Bonnans, D. Giorgi, B. Heymann, P. Martinon, and O. Tissot, "BocopHJB 1.0. 1--User Guide,", INRIA, 2015.

[25] R. Carnegie, D. Gotham, D. Nderitu, and P. V. Preckel, "Utility scale energy storage systems," State Utility Forecasting Group, Purdue University, vol. 1, 2013.

[26] A. Y. Petrov, A. Zaltash, S. D. Labinov, D. T. Rizy, and R. L. Linkous, "Dynamic performance of a 30-kW microturbine-based CHP system," *Ashrae Trans.*, vol. 111, no. 1, pp. 802-809, 2005.

[27] "Dataport," Pecan Street Inc., [Online]. Available: https://dataport.cloud/, [Accessed: 20th Jul. 2018].

[28] B. Heymann, *et al.*, "Continuous optimal control approaches to microgrid energy management," *Energy Systems*, pp. 1-19, 2015.



Yuchao Qin was born in Henan, P. R. China in 1994. He received the B.Sc. degree in automation in 2017 from Tsinghua University, Beijing, P. R. China.

He is currently a postgraduate student in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include control and optimization, machine learning and their applications in power system.

Haochen Hua (M'16) was born in Jiangsu, P. R. China in 1988. He received the B.Sc. degree in mathematics with finance in 2011, and the Ph.D. degree in mathematical sciences in 2016, both from the University of Liverpool, Liverpool, UK.

He is currently a post-doctoral researcher in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include optimal and robust control theory and its applications in power systems, smart grids, and the energy internet.

Junwei Cao (SM'05) received his Ph.D. in computer science from the University of Warwick, Coventry, UK, in 2001. He received his bachelor and master degrees in control theories and engineering in 1998 and 1996, respectively, both from Tsinghua University, Beijing, China. He is currently Professor and Vice Dean of Research Institute of Information Technology, Tsinghua University, Beijing, China. He is also Director of Open Platform and Technology Division, Tsinghua National Laboratory for Information Science and Technology.

Prior to joining Tsinghua University in 2006, he was a Research Scientist at MIT LIGO Laboratory and NEC Laboratories Europe for about 5 years. He has published over 200 papers and cited by international scholars for over 18,000 times. He has authored or edited 8 books. His research is focused on distributed computing technologies and energy/power applications.

Prof. Cao is a Senior Member of the IEEE Computer Society and a Member of the ACM and CCF.