Voltage Regulation in Edge Energy Router System via H_∞ Control with Markov Jump

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Abstract—With the increase of the amount of tasks offloaded to the network edge, the energy supply of edge devices has become a challenge worthy of attention. It is a feasible way to use renewable energy to supply energy for edge devices, but the production of renewable energy has certain uncertainty and stochasticity. In order to provide sufficient energy to ensure the stable operation of edge devices, energy Internet (EI) provides an idea, that is, different edge devices are connected with distributed small energy supply and storage systems. As the core equipment of energy Internet, energy router (ER) plays an important role in information transmission, energy transmission and system control. In this paper, the concept of edge energy router is proposed, which has the ability of task computing and scheduling similar to edge computing server, as well as the ability of energy transmission and system control of energy router. Each edge energy router is connected with loads, photovoltaic panel (PV), micro turbine (MT) and battery energy storage (BES) to form a self-sufficient microgrid (MG) system. However, there exists delay in energy transmission and task scheduling between different ERs. Moreover, the DC bus voltage stability of each edge energy router system is negatively affected by internal uncertainty, stochasticity and external interference. Therefore, the system is modeled by Markov jump ODEs with time delay, and the robust control of DC bus voltage deviation is discussed in this paper. The linear matrix inequality (LMI) method is used to solve this markov jump control problem. Finally, numerical simulations show the effectiveness of the proposed method.

Index Terms—edge computing, edge energy router, microgrid, markovian jump system, ordinary differential equation

Nomenclature

Abbreviations

EC	Edge Computing
EI	Energy Internet
IoT	Internet of Things
ER	Energy Router
PV	Photovoltaic panel
WT	Wind turbine
MT	Micro turbine
BES	Battery energy storage
MG	Microgrid
LMI	Linear matrix inequality
ODE	Ordinary differential equation
MEC	Multi-access Edge Computing
DOS	Denial of Service

Variables

ΔP_{PV}	Output power changes of PV
ΔP_{MT}	Output power changes of MT
ΔP_{EE}	Energy exchange changes of edge ER
ΔP_L	Power changes of loads
ΔP_{BES}	Charging/Discharging power of BES
ΔS_{ED}	Changes of computing task of edge ER
ΔS_{TS}	Changes of task scheduling of edge ER
Δq_{TS}	Changes of average queue length
ΔV	DC bus voltage deviation
ΔP	Changes of bus power of edge ER

Constants

n	The number of edge energy routers
S	State space of Markov chain
T_{TC}	Time constant of edge energy routers
T_L	Time constant of loads
T_{PV}	Time constant of PVs
T_{MT}	Time constant of MTs
T_{BES}	Time constant of BESs
T_{EE}	Time constant of energy exchange changes
b_{MT}	System coefficient of MTs
b_{EE}	System coefficient of EE
r_{BES}	System parameter of BESs
p	System parameter of voltage deviation
q	System parameter of voltage deviation
N_0	Number of task transmission lines
R_0	Round trip time
C_0	Task transmission capacity
γ	Disturbance rejection ability factor
τ	Time delay of power exchange

I. Introduction

A. Background

Recently, the rapid development of the Internet of things (IoT) has led to the deployment of massive end devices such as sensors [1]. These IoT devices continue to produce a large amount of data, making the human society into the era of big data [2], [3]. The analysis and utilization of these massive data require sufficient computing and storage resources [4]. In order to meet this demand, the conventional method is to offload all these data to the cloud for processing and storage [6], [5]. However, directly offloading massive data to the cloud may lead to insufficient network bandwidth and high transmission delay [7], [8]. In order to solve these problems, EC, as a new computing paradigm, provides a new idea. EC refers to offload all or part of the tasks originally stored and processed in the cloud to the edge side closer to the end device [9]. By doing this, lower transmission delay and higher quality of service can be achieved [10].

According to the prediction of the white paper [11], by 2025, the number of IoT devices in the world will increase from 1.84 billion in 2018 to 2.93 billion, and 63% of the computing tasks generated will be performed on the network edge. With the increasing amount of tasks performed by the edge server, the energy consumption of edge server is a problem worthy of concern. According to the previous discussion, as the number of edge servers and the tasks performed by the edge side increase, the energy consumption would be very high [12]. In order to process the large amount of computing tasks of EC, the additional energy consumption is 30-50% [15]. With the aggravation of global warming, environmental pollution and the depletion of fossil fuels, renewable energy, as an alternative to fossil energy, has been widely concerned by industry and academia [14].

In this context, a kind of energy system concept, microgrid (MG), which can make full use of renewable energy, is just suitable for solving the energy supply problem of EC system [15]. In [15], the microgrid-enabled multi-access edge computing (MEC) networks' energy supply plan is studied to minimize the energy consumption of this MEC system. In [12], a energy management framework for enabling a sustainable and green EC paradigm powered by MG. The proposed framework aims to make full use of renewable energy and ensure the quality of EC services. Similarly, the author in [16] combines MG with EC to save energy consumption.

Considering that single MG may have irreparable power deviation due to the uncertainty of renewable energy, energy exchange between multiple MGs has been widely concerned [17]- [19]. Many researches focus on voltage and frequency stability control and optimization in MG. In [20], a bottom-up energy management issue, which is formulated into a stochastic optimal control problem, is solved by dynamic programming approach. A novel secondary control strategy is proposed in [21] for the power-electronic-based MG to maintain the stability of voltage and frequency deviation. In addition, one of the important issues in MG is to extend the service life of energy storage devices [22]- [30]. In order to obtain more accurate photovoltaic and load energy model, a method combining neural network and stochastic differential equation is proposed in [23]. And a method to prolong the battery life is proposed.

However, there are not only obvious stochasticity and uncertainty in the energy system [26], such as photovoltaic power generation is directly affected by the intensity of sunlight, and the load is affected by the life activities of local users, but also stochasticity and uncertainty in the edge system [25], such as the number of computing tasks and the current resource utilization of computing equipment. In order to simulate the real stochasticityc and uncertainty in system modeling, some classical methods, such as Brownian motion, white noise method and so on, have been applied in many researches [27]. Markov jump system is a dynamic system controlled by one of a group of linear systems, and it is determined by a continuous time Markov process which linear system is active. When the current state of Markov process changes, the dynamics of continuous state "jumps" from one system to another [24]. Markov jump system has been applied in many researches on control problems of MG. In [28], markov jump is used to describe the stochasticity of time delay in MG system. In order to keep the frequency stability of MG under random DoS attack, in [29], Markov jump system is used to describe the different states of the system under different attacks.

With the rapid development of information technology, energy Internet (EI), as a feasible solution, can manage the energy flow exchange of multiple MGs more efficiently [31]- [33]. ER is the core device for energy and information exchange between different MGs [34], [35]. Recently, research on energy routing strategies and energy management problem for ERs has attracted much attention [36]. In order to maintain voltage stability in ER system, an H_∞ control method is proposed in [36]. ER can access and manage PV, BES, MT and other devices [38], [39]. Therefore, it is reasonable to believe that it is a feasible method to provide energy for edge devices through ER system. There are many research work on the energy supply of edge devices through renewable energy [13], [15]. However, it is worth noting that there has been few work focusing on the power supply of edge devices through ER system.

B. Motivation

According to the above discussion, the motivation of proposing edge ER mainly includes the following two points. Firstly, with the increase of the number of tasks offloaded to edge devices, the power supply of edge devices has become a noteworthy challenge [9]. Considering that the geographical distribution of edge devices is decentralized, it is a feasible way to use local renewable energy to provide energy. As a self-sufficient local MG, ER system can be used to maintain the energy supply of task computing in edge devices [12]. In addition, as the information and energy management center in ER system, ER needs certain computing and storage capacity, which is similar to edge devices. Therefore, considering the above two points, we propose a new concept called edge ER, which not only has the ability of EC, but also is responsible for the information and energy management in the ER system. Its features include the following points: 1) It has the basic function of EC devices, and has powerful computing power and storage capacity, so it can undertake part of the computing tasks of cloud server. 2) Multiple different edge ERs have the ability of communication and can schedule tasks among each other. 3) It has the function of power transmission to realize the energy complementary between different edge ERs. 4) Each edge ER is connected with PV, MT, load and BES, which has a certain self-sufficiency.

In order to ensure that the energy required for task computing is provided, it is very important to keep the DC bus voltage deviation stable in each edge ER system. H_∞ control is to suppress the maximum gain of the transfer function set from noise to expected output, so as to achieve the purpose of disturbance rejection. For the term H_∞, the letter H represents Hardy space and the symbol ∞ represents infinite norm. The main goal of this paper is to find a state feedback controller, so that the output of the system, that is, the DC bus voltage deviation, remains stable under various external disturbances. H_∞ just transforms this kind of problem into an optimization problem, so that we can solve this problem and obtain an effective state feedback controller. Therefore, a robust H_∞ control problem is proposed to provide energy for the task computing of edge ER system and regulate the stability of DC bus voltage in this paper. Moreover, considering the stochasticity and uncertainty of the connected PVs, MTs, loads and BES, and in order to better express these characteristics, Markov jump ODEs is used to model this system. It is worth noting that in this paper, information scheduling and energy transmission between different edge ER also absorb DC bus voltage deviation. Then, the above problem is transformed into an H_∞ robust control problem with Markov jump, and finally solved by an LMI method.

C. Contribution

The main contributions of this paper are as follows:

1) The concept of edge ER is proposed for the first time. This not only makes the computing and scheduling of information tasks and energy management better combined, which is convenient for unified control, but also energy management can be supported by computing resources of EC. Moreover, in order to better express the dynamic of edge ER system, ODEs are used to model the PVs, loads, BESs, MTs, task scheduling, energy transmission, and DC bus voltage deviation. It is worth noting that this is the first time that the task scheduling and computing of EC and the energy supply of ER system are considered simultaneously,

2) This is the first time that Markov jump ODEs are used not only to model energy system, but also to model information scheduling and task computing. The edge ER system, a newly proposed physical power system, is modelled as a system with Markovian jump, external disturbance and time delay. By doing this, the modelling of this system is closer to the real physical scene, and the stochasticity of the system can be better expressed.

3) The problem of DC bus voltage regulation in edge ER system is formulated as problems of robust stabilization and robust H_∞ control of Markov jump system. Although the modelled system is complex, an explicit solution can be found in a state feedback form, which is easy and flexible to be implemented in real engineering practice. The results show that the proposed method has a good effect on the stability of DC bus voltage deviation in the edge ER system mentioned in this paper.

4) On the basis of satisfying LMI, we further improve the effect of the control method for stabilizing voltage deviation by minimizing γ. Simulation results show that the proposed H_∞ method has better control effect than the traditional H_∞ method, and can deal with different forms of external disturbances.

The rest of the paper is organized as follows. Section II describes the system architecture and the modeling of PV, loads, edge devices, BES, MT and energy exchanging. And a mathematical state space control system is formulated. Section III formulates the robust control problem and provides the solutions. In Section IV, some numerical simulations and summarizes is provided. Finally, the conclusion is presented in Section V.

II. System Modeling

In this section, an ER system architecture for powering EC system is introduced and modelled via a class of ordinary differential equations (ODEs).

A. Description of System

In order to generalize the edge ER system, it is assumed that there are n edge ERs, and there could exists some connections between two edge ERs. The connection between two edge ERs means that there is energy exchange between them.

As shown in Fig. 1, a example of ER system architecture is described. Due to the distance, geographical location and other reasons, not all edge ERs are connected. On the left side of Fig. 1, in the physical layer, multiple edge ERs are connected by energy transmission lines, in which the blue line represents the energy transmission line. In addition, in the information layer, there is information transmission between some edge ERs, which is indicated by yellow lines. The detailed composition of a single edge ER is shown on the right side of Figure 1. The ER system usually connects PV, MT, BES and various loads. It is worth noting that the edge ER studied in this paper has the function of EC as well as energy management. The

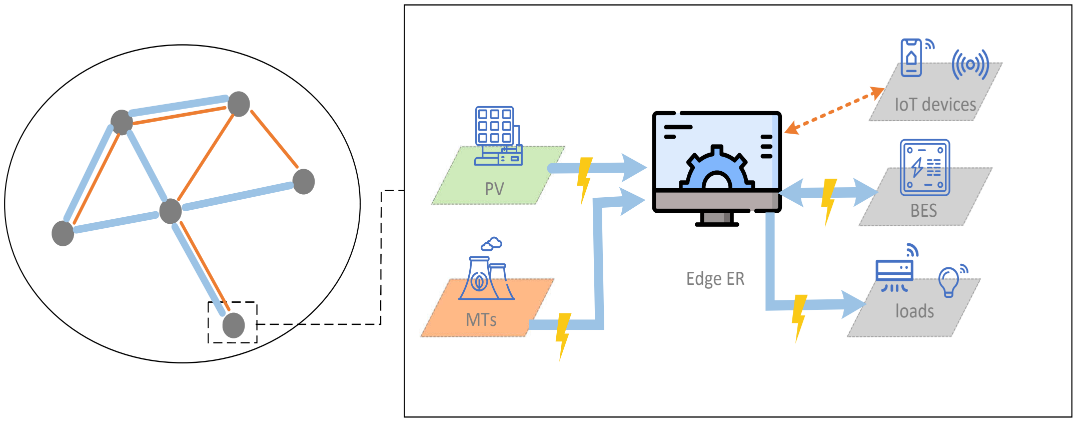


Fig. 1. The architecture of edge ER system

orange dotted line indicates the device information and task scheduling between the router and IoT.

An undirected graph denoted as $G(V, \xi)$ is utilized to describe different edge ERs and connections between them in the physical layer. The V is represented by n different edge ERs, and the ξ is represented by the connections between edge ERs. The l-th edge of an undirected graph connects the i-th and j-th nodes, which means that energy can be transmitted between the i-th and j-th edge ERs. If an ER cannot be self-sufficient, it can obtain energy from other connected edge ERs through energy transmission.

In order to make the system modelling closer to the real physical scene, and better express the stochasticity of the system, the state transition of system parameters follows a finite-state Markov jump process in this paper. Markov chain r_t^n is n-th finite-state Markov jump process representing the studied ER system mode. The state space of Markov chain is $S = 1, 2, ..., s$. The transition probability matrix is denoted as $\Pi = [\pi_{i,j}]$, where $i, j = 1, 2, ..., s$, and satisfies

$$Pr\{r(t+dt) = j | r(t) = i\} = \begin{cases} \pi_{i,j} dt, & i \neq j \\ 1 + \pi_{i,i} dt, & i = j \end{cases} \quad (1)$$

where $\pi_{i,j} \geq 0$, $\pi_i \triangleq \pi_{i,i} \leq 0$ with $\sum_{j=1, j\neq i}^{s} \pi_{i,j} = -\pi_{i,i}$.

B. Modeling of Task Computing and Scheduling

In this paper, edge ER has the functions of both edge device and ER. On the one hand, it can process and schedule computing tasks. On the other hand, it is responsible for controlling the energy transmission and connected energy devices, such as MT, according to the power supply and demand. Task computing and task scheduling have an important impact on the energy consumption of the edge ER system studied in this paper. To describe these effects dynamically, ODEs are used to model task computation and task scheduling.

With the development of smart city, more and more computing tasks put forward strict requirements for delay and network stability. As an effective and promising computing paradigm, EC is responsible for a part of the computing tasks that were originally processed in the cloud. So the energy consumption caused by these massive tasks can not be ignored for the edge ER considered in this paper. Considering the dynamic nature of EC, the resource allocation of EC system is modeled by ODE in [40]. Therefore, in order to better express the dynamic nature of task computing, we also use ODEs to express the amount of computing tasks . In this paper, it is considered that the edge ERs may be disturbed by external interference, such as the sudden increase of task requests from users. These disturbances could significantly change the energy consumption of edge devices, which would affect the DC bus voltage deviation. Considering all the above factors, the changes of computing task in the k-th edge ER is modeled by an ODE with interference [6].

$$\Delta \dot{S}_{TC}^k = -\frac{1}{T_{TC}^k(r_t^n)} S_{TC}^k + \frac{1}{T_{TC}^k(r_t^n)} v_{TC}^k, \quad (2)$$

where $T_{TC}^k(r_t^n)$ is the time constant of edge ERs, and v_{TC}^k represents the external interference, such as the sudden increase of task requests from users.

Considering that the computing and energy supply capacity of each edge ER is different at different time points, the task scheduling between different edge ERs is the guarantee to ensure the smooth processing of computing tasks. Here, TCP is used as the network transport protocol of the studied edge ER system. A mathematical dynamic equation of TCP based on ODE is proposed in [41]. And the modeling method has been proved to be correct and effective by different applications and simulations [42]. In this paper, the nonlinear TCP model in [41] is used to describe the task transmission between different edge ERs.

The model is as follows.

$$\Delta\dot{S}_{TS}^{m} = \frac{1}{R^m} - \frac{W^m W^m(t-R^m)}{2R^m} u_{TS}(t-R^m), \quad (3)$$

$$\dot{q}_{TS}^{m} = \frac{W^m N^m}{R^m} - C^m \quad (4)$$

where $\Delta\dot{S}_{TS}^{m}$ denotes the change in the amount of tasks scheduled on the m-th communication line and $\dot{q}_{TS}^{m}$ is the average queue length. R^m denotes the round-trip time for information transmission on the m-th communication line. W^m and C^m are respectively the number of information transmission line and the information transmission capability of the considered communication line. By adjusting the control signal u_{TS}, the amount of task can be adjusted accordingly. When the task quantity of an edge ER is too large, more tasks are scheduled to other routers by setting a larger value to the control signal.

This paper focuses on the DC bus voltage stability of edge ER system. Therefore, according to the [42], the nonlinear TCP model is simplified as linear ODEs. In addition, in the process of task transmission, the uncertainty of model parameters is also considered. Therefore, the simplified linear formula is further expressed as ODEs with Markov jump parameters. The task scheduling model with Markov jump parameters is shown as follows.

$$\Delta\dot{q}_{TS}^{m} = \frac{N_0^m(r_t^n)}{R_0^m(r_t^n)}\Delta\dot{S}_{TS}^{m} - \frac{1}{R_0^m(r_t^n)}q_{TS}^{m} \quad (5)$$

$$\begin{aligned}\Delta\dot{S}_{TS}^{m} = &-\frac{2N_0^m(r_t^n)}{R_0^m(r_t^n)C_0^m(r_t^n)^2}(\Delta\dot{S}_{TS}^{m} + \Delta\dot{S}_{TS}^{m}(t-R_0^m(r_t^n)))\\ &-\frac{1}{R_0^m(r_t^n)C_0^m(r_t^n)^2}(\Delta q_{TS}^{m} - \Delta q_{TS}^{m}(t-R_0^m(r_t^n)))\\ &-\frac{R_0^m(r_t^n)C_0^m(r_t^n)^2}{2N_0^m(r_t^n)^2}u_{TS}^{m} \quad (6)\end{aligned}$$

where $N_0^m(r_t^n)$, $R_0^m(r_t^n)$, $C_0^m(r_t^n)$ denote the number of task transmission lines, round trip time and task transmission capacity, respectively. u_{TS}^{m} and $\Delta\dot{S}_{TS}^{m}$ the transmission control signal and changes of the amount of tasks scheduled on the m-th transmission line. The $\Delta\dot{q}_{TS}^{m}$ is the the average queue length on the m-th transmission line.

C. Modeling of Loads and PV

In order to provide energy for edge devices more environmentally friendly and economically, it is necessary to make full use of renewable energy. In this paper, PV is used as the source of renewable energy. Considering that PV and loads could be subject to external interference, such as the sudden change of light intensity and power using of users, and in order to more truly model loads and PV, the ODEs with disturbance are used to model them. The power changes of loads and PV in the k-th ER are modeled as follows (time t is omitted) [45].

$$\Delta\dot{P}_{L}^{k} = -\frac{1}{T_L^k(r_t^n)}\Delta P_L^k + \frac{1}{T_L^k(r_t^n)}v_L^k, \quad (7)$$

$$\Delta\dot{P}_{PV}^{k} = -\frac{1}{T_{PV}^k(r_t^n)}\Delta P_{PV}^k + \frac{1}{T_{PV}^k(r_t^n)}v_{PV}^k, \quad (8)$$

where $T_{PV}^k(r_t^n)$ and $T_L^K(r_t^n)$ are the time constant of loads and PV respectively, and v_L^k and v_{PV}^k represent the external interferences, such as the sudden change of light intensity and power using of users.

D. Modeling of MT

Renewable energy has strong stochasticity, which may lead to the imbalance of power supply-demand in edge ER, which seriously affects the normal work of edge devices and other loads. Therefore, MT, which consumes traditional fossil, is deployed in each edge ER as a controllable power supply in this paper. By controlling the power generation of MT, the shortage of PV power generation can be compensated. The power changes of MT in the k-th edge ER is modeled as follows [45].

$$\Delta\dot{P}_{MT}^{k} = -\frac{1}{T_{MT}^k(r_t^n)}\Big(\Delta P_{MT}^k + b_{MT}^k(r_t^n)u_{MT}^k\Big), \quad (9)$$

where $T_{MT}^k(r_t^k)$ is the time constant of MT, $b_{MT}^k(r_t^k)$ is the system coefficient related with the controller, and u_{MT}^k represents the control input.

E. Modeling of Power Exchange

And a controller is designed to control the change of power exchange, so as to keep the balance of power supply-demand of the whole edge ER system. The modeling of power exchange between in l-th energy transmission line ΔP_{EE}^{l} is as follows.

$$\Delta P_{EE}^{l} = \frac{1}{T_{EE}^l(r_t^n)}\Big[-P_{EE}^l(t-\tau) + b_{EE}^l(r_t^n)u_{EE}^l\Big], \quad (10)$$

where $T_{EE}^k(r_t^n)$ is the time constant of power exchange, u_{EE}^l control input of power exchange, and $b_{EE}^l(r_t^n)$ represents the system coefficient related with the controller.

F. Modeling for BES and Voltage Deviation

Energy storage equipment can maintain the power supply-demand balance of edge ER by absorbing power deviation, so as to ensure the normal work of edge devices. Therefore, The BES is deployed in every edge ER in this paper. The modeling of BES is as follows [45].

$$\Delta\dot{P}_{BES}^{k} = -\frac{1}{T_{BES}^k(r_t^n)}\Big(\Delta P_{BES}^k + r_{BES}^k(r_t^n)\Delta V^k\Big), \quad (11)$$

where $T_{BES}^k(r_t^n)$ is the time constant of BES, and $r_{BES}^k(r_t^n)$ represents the system parameter. Since the research focus is not on the BES itself, the state of charge (SOC) is not considered in this paper. The similar modeling has been found in, e.g., [43] and [44]. ΔV^k is the DC bus voltage deviation which is modeled as follows [36].

$$\Delta\dot{V^k} = -\frac{1}{p^k(r_t^n)}\Delta V^k + -\frac{1}{q^k(r_t^n)}\Delta P^k, \tag{12}$$

where $p^k(r_t^n)$ and $q^k(r_t^n)$ are the system parameters. ΔP^k is denoted the bus power of the k-th edge ER, and satisfies the following equation.

$$\Delta P^k = -E\cdot\Delta S_{TC}^k - \Delta P_L^k + \Delta P_{PV}^k + \Delta P_{MT}^k$$

$$\pm\Delta P_{BES}^k + \sum_{l=1}^{L} f(k,l)\cdot\Delta P_{EE}^l \tag{13}$$

$$+\sum_{m=1}^{M} g(k,m)\cdot E\cdot\Delta S_{TS}^l. \tag{14}$$

The function $f(k,l)$ represents the direction of energy exchange. When $f(k,l)=1$, the k-th edge ER transmits energy flow to other edge ER through the line l. When $f(k,l)=-1$, the k-th edge ER receives the energy transmitted from other edge ER through the line l. L means there are totally L energy transmission lines. Similar, M and $g(k,m)$ represent totally M information communication lines and the direction of information communication, respectively. E is the proportional relationship between task changes and power consumption changes.

G. Modeling for Edge ER System

In this subsection, we simplify the system into a mathematical state space control system. Firstly, we define the state vector of k-th edge ER as $x^k = [\Delta S_{TC}^k, \Delta P_{PV}^k, \Delta P_L^k, \Delta P_{MT}^k, \Delta P_{BES}^k, \Delta V^k]'$. The control input vector is u_{MT}^k, and disturbance input vector is defined as $v^k = [v_{TC}^k, v_{PV}^k, v_L^k]'$. The system state vector of transmission line is defined as $x_{EE} = [\Delta P_{EE}^1, \Delta P_{EE}^2, ..., \Delta P_{EE}^l]'$. The system state vector of communication line is defined as $x_{TS} = [\Delta S_{TS}^1, \Delta S_{TS}^2, ..., \Delta S_{TS}^m, \Delta q_{TS}^1, \Delta q_{TS}^2, ..., \Delta q_{TS}^m]'$. The control input vector of transmission line is defined as $u_{EE} = [u_{EE}^1, u_{EE}^2, ..., u_{EE}^l]'$. The control input vector of communication line is defined as $u_{TS} = [u_{TS}^1, u_{TS}^2, ..., u_{TS}^m]'$.

And then, the total system state vector can be obtain, which is $x = [x^{1'}, x^{2'}, ..., x^{n'}, x_{EE}^{'}, x_{TS}^{'}]'$. The total control vector is $u = [u_{MT}^{1'}, u_{MT}^{2'}, ..., u_{MT}^{n'}, u_{EE}^{'}, u_{TS}^{'}]'$. The output vector of this system is $z = [\Delta V^1, \Delta V^2, ..., \Delta V^n]'$.

By doing this, the system can be transformed into a mathematical state space control system.

$$\begin{cases} dx = [A(r_t)x + Ad(r_t)x(t-\tau) + B(r_t)u + C(r_t)v]dt \\ z = D(r_t)x \end{cases} \tag{15}$$

III. Problem Formulation and Solution

In this section, the problem of stabilizing voltage for edge ER system is formulated as a robust H_∞ control problem. Through this robust H_∞ control method, the bad influence of external disturbance on the stability of DC bus voltage deviation in the edge ER system can be suppressed by adjusting the gamma value. Then, this control problem is solved analytically.

In fact, the considered edge ER system has been affected by various external disturbances. For example, the power change of load caused by sudden access to high-power electrical equipment, the power change of PVs caused by the change of light intensity, and the power change of calculation energy consumption caused by sudden increase of user task request, etc.

Therefore, the main goal of this paper is to design a robust controller to keep the DC bus voltage in the edge ER system stable under external disturbance. In order to achieve this goal, we introduce a scalar γ which represents the disturbance rejection ability of voltage stability. The smaller the γ is, the stronger the anti-interference ability of the controller is and the more robust the system is.

Definition 1: Given a scalar $\gamma > 0$, the H_∞ performance of the DC bus voltage robust control problem proposed in this paper is $\|z(t)\| < \gamma\|v(t)\|$, where $\|\cdot\|$ is defined as follows.

$$||z(t)|| \triangleq \left(\mathbb{E}\left\{\int_0^\infty |z(t)|^2\, dt\right\}\right)^{\frac{1}{2}}, \tag{16}$$

where $\mathbb{E}$ represents for the mathematical expectation. Based this H_∞ performance, the cost functional is formulated as follows.

$$J(u,v) \triangleq \mathbb{E}\left[\int_0^T (\gamma^2 v'v - z'z)dt\right]. \tag{17}$$

The H_∞ control problem is to find a controller u^*, such that for all nonzero disturbance v, $J(u,v) \le 0$ holds. According to (17), it is obvious that the smaller the γ, the stronger the interference suppression ability of the controller. By using the method proposed by [46], a theorem is obtained to solve this problem.

Theorem 1. If there exist symmetric matrices $X_i > 0$, $R > 0$ and matrix Y_i satisfying following LMI. A controller to stabilize the system output can be formulated as $K_i = Y_i X_i^{-1}$ and $u_i^* = -K_i x$.

$$\begin{bmatrix} \Omega_i + Ad_i R Ad_i' & X_i' D' & C_i & \Xi_i \\ DX_i & -I & 0 & 0 \\ C_i' & 0 & -\gamma_2 I & 0 \\ \Xi_i' & 0 & 0 & -\Gamma_i \end{bmatrix} \le 0 \tag{18}$$

where $\Omega_i \triangleq X_i A_i' + A_i X_i - Y_i' B_i' - B_i Y_i + \pi_i X_i$,
$\Xi_i \triangleq [\sqrt{\pi_{i,1}}X_i, \cdots, \sqrt{\pi_{i,i-1}}X_i, \sqrt{\pi_{i,i+1}}X_i, \cdots, \sqrt{\pi_{i,s}}X_i, X_i]$ and $\Gamma_i \triangleq diag[X_i, \cdots, X_{i-1}, X_{i+1}, \cdots, X_s, R]$. To simplify the notation, $A(r_t)$ is denoted by A_i, and so no.

Proof: The proof idea is from [46]. Let us denote $\hat{A}_{1i} = A_i - B_i K_i$. By introducing $u_i = -K_i x$ into system (15), the closed-loop system (19) is obtained.

$$\begin{cases} dx = [\hat{A}_{1i}x + Ad_i x(t-\tau) + C_i v]dt \\ z = D_i x \end{cases} \tag{19}$$

In this paper, we choose the stochastic Lyapunov functional V(x,i) shown as follows:

$$V(x,i) = x'(t)P_i x(t) + \int_{-\tau}^{0} x'(t-\theta)Qx(t-\theta) \tag{20}$$

Then, we have

$$\begin{aligned} AV(x,i) =& X_i'[\hat{A}_i' p_i + P_i\hat{A}_i + \sum_{j=1}^{s}\pi_{ij}p_j]X_i \\ &+ X_i'(t-\tau)Ad_iP_iX_i + X_i'P_iAd_iX_i(t-\tau) \\ &+ v'C_i'P_iX_i + X_iP_iC_iv + X_i'QX_i \\ &- X_i'(t-\tau)QX_i(t-\tau). \end{aligned} \tag{21}$$

In the following, based on Dynkin's formula [47] and the assumption of zero initial condition , we have

$$EV(x,i) = E\{\int_0^T AV(x(s), i(s))ds\} \tag{22}$$

So, the cost functional can be rewritten as:

$$\begin{aligned} J_{u,v} &= E\{\int_0^T [z'z - \gamma^2 v'v + AV(x,i)]dt\} - EV(x,i) \\ &\le E\{\int_0^T [z'z - \gamma^2 v'v + AV(x,i)]dt\} \\ &= E\{\int_0^T [\delta'\Theta_i\delta]dt\} < 0, \end{aligned} \tag{23}$$

where $\delta = [x', x'(t-\tau), v']'$ and Θ_i is shown as follow: .

$$\Theta_i = \begin{bmatrix} \hat{A}_{1i} & P_iAd_i & P_iC_i \\ Ad_i'P_i & -Q & 0 \\ C_i'P_i & 0 & -\gamma_2 I \end{bmatrix} \le 0 \tag{24}$$

where $\hat{A}_{1i} = \hat{A}_i'P_i + P_i\hat{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + Q + D_i'D_i$. From the Schur complement [46] , we can obtain the following matrix inequalities based on (24).

$$\begin{bmatrix} \hat{A}_i'P_i + P_i\hat{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + Q & P_iAd_i & P_iC_i & D_i' \\ Ad_i'P_i & -Q & 0 & 0 \\ C_i'P_i & 0 & -\gamma_2 I & 0 \\ D_i & 0 & 0 & -I \end{bmatrix} \le 0 \tag{25}$$

By pre- and postmultiplying (25) by $T_i = diag(X_i, I, I, I)$, the matrix inequalities are equivalent to the following matrix inequalities:

$$\begin{bmatrix} \Omega_i & Ad_i & C_i & X_iD_i' \\ Ad_i' & -Q & 0 & 0 \\ C_i' & 0 & -\gamma_2 I & 0 \\ D_iX_i & 0 & 0 & -I \end{bmatrix} \le 0 \tag{26}$$

where $\Omega_i = X_i\hat{A}_i' - Y_iB' + \hat{A}_iX_i + X_i\sum_{j=1}^{s}\pi_{ij}P_jX_i + X_iQX_i$. From the Schur complement, we obtain the matrix inequalities as follows. And the LMI (18) can be obtained.

$$\begin{bmatrix} \Omega_i + Ad_iQ^{-1}Ad_i' & X_iD_i' & C_i \\ D_iX_i & -I & 0 \\ C_i' & 0 & -\gamma_2 I \end{bmatrix} \le 0 \tag{27}$$

The proposed edge ER system with Markov jump is translated to a dynamic system controlled by one of a group of linear systems, and it is determined by a continuous time Markov process which linear system is active. When the current state of Markov process changes, the dynamics of continuous state "jumps" from one system to another. In order to find the control input $u_i^* = -K_ix$ in different Markov states, it is necessary to solve the LMI corresponding to each group of systems, that is, formula 18 in Theorem 1.

IV. Numerical Simulation

In this section, we solve this H_∞ control problem by a LMI method. Numerical simulation results are provided to verify the feasibility of the proposed method in this section.

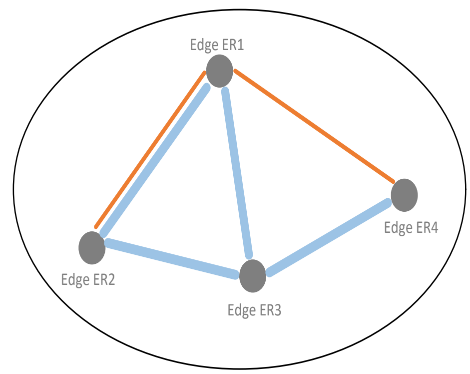


Fig. 2. A typical edge ER system for numerical simulation

For the convenience of simulation, a system composed of four edge ERs is shown in Fig. 2, in which the blue line and orange line indicate the existence of energy transmission and information task scheduling respectively. In order to be more practical, considering the limitations of communication and geographical location, not every two edge ERs have information or energy transmission in the edge ER system considered in this paper.

The system parameters are listed in the Table I. It is assumed that the system parameter transition for all edge ERs follows the same Markov process. And the number of system mode is set five, which is $S = [1, 2, ..., s]$ and $s = 5$. The system parameters are measured by parameter estimation method [44]. The system parameters in different modes are randomly generated by taking the values in TABLE I as mean values and following uniform distribution.

Case 1. In this case, under the control of the proposed H_∞ control method, the stability of DC bus voltage deviation is demonstrated. As can be seen in Fig. 3, the state mode of the system changes from 1 to 5. The solution of the Markov jump system control problem is essentially equivalent to finding the corresponding control signal u^* in different state modes. In this paper, we got 5 different

TABLE I
Time Constants of The ER System

Parm	Value	Parm	Value	Parm	Value
T_{TC}^1	1.7	T_{TC}^2	1.3	T_{TC}^3	1.5
T_{TC}^4	1.8	T_{PV}^1	1.6	T_{PV}^2	1.2
T_{PV}^3	1.4	T_{PV}^4	1.7	T_L^1	1.0
T_L^2	1.3	T_L^3	1.5	T_L^4	1.4
T_{MT}^1	0.05	T_{MT}^2	0.03	T_{MT}^3	0.04
T_{MT}^4	0.06	T_{BES}^1	0.5	T_{BES}^2	0.7
T_{BES}^3	0.3	T_{BES}^4	0.6	T_{ER}^1	0.14
T_{ER}^2	0.10	T_{ER}^3	0.09	T_{ER}^4	0.13
b_{MT}^2	1.0	b_{MT}^3	1.4	b_{MT}^4	1.2
r_{BES}^1	1.1	r_{BES}^2	1.2	r_{BES}^3	1.4
r_{BES}^1	1.0	b_{ER}^1	1.5	b_{ER}^2	1.3
b_{ER}^3	1.1	b_{ER}^4	1.0	p^1	0.12
p^2	0.10	p^3	0.14	p^4	0.13
q^1	0.030	q^2	0.033	q^3	0.027
N_0^1	10	N_0^2	12	R_0^1	0.20
$R0^2$	0.24	C_0^1	1500	C_0^2	1700
q^4	0.036	γ	0.8	E	0.8
T	10	k	0.01	$\pi_{i,j}$	0.001

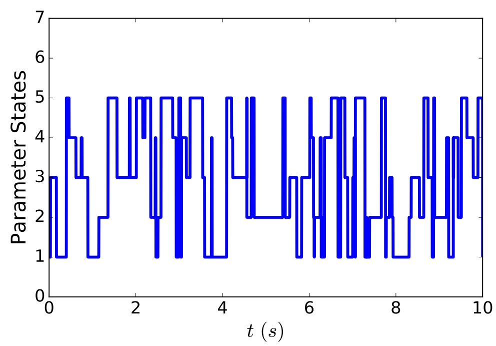


Fig. 3. State transition of system parameters.

controller u^*, which are u^1, u^2, u^3, u^4 and u^5 respectively. For example, when the state mode changes from 2 to 1, the control signal also changes from u^2 to u^1.

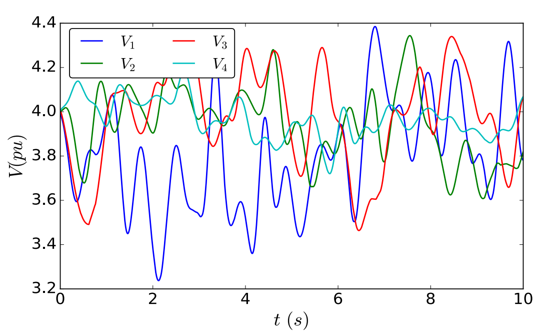


Fig. 4. DC bus voltage deviation without control

The main purpose of this paper is to keep the internal DC bus voltage deviation of each edge ER stable under

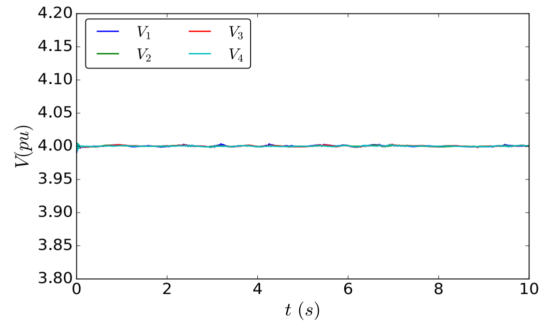


Fig. 5. DC bus voltage deviation with control

the external disturbance and system parameter jump. Therefore, it is reasonable to assume that the DC bus voltage deviation is stable before the initial time. The initial DC bus voltage value is set to 4. In Fig. 4, the DC bus voltage deviation without proposed H_∞ control is obviously unstable under the continuous disturbance. By comparing Fig. 4 and Fig. 5, it is obvious that the proposed method has good performance in suppressing external disturbance. The value of DC bus voltage always fluctuates around 4 after stabilization.

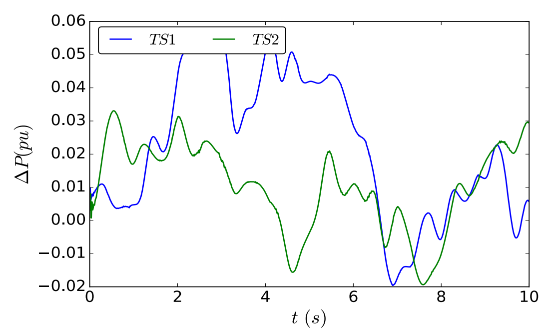


Fig. 6. Power of task transmission with control

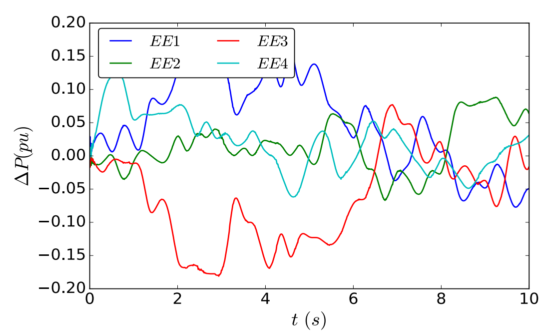


Fig. 7. Power of energy exchange with control

The energy changes caused by information scheduling and energy transmission under the proposed H_∞ control method with Markov jump are shown in Fig. 6 and Fig. 7 respectively. According to Fig. 5, Fig. 6 and Fig. 7, it can be inferred that the energy transmission and

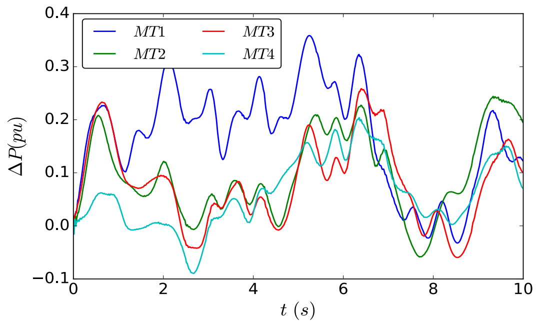


Fig. 8. Power of MT with control

task scheduling between different edge ERs can effectively reduce the influence of external interference and system parameter jump on DC bus voltage deviation under the control of proposed H_∞ control method.

Due to the stochasticity of load and PV power generation in the edge ER system, MT needs to change its power generation under the control of the controller, as shown in Fig. 8. It can show that part of the DC bus voltage deviation can be absorbed by MTs.

parameters caused by Markov jump, so as to ensure the control effect.

In Fig. 9, the control effect of voltage stability in edge ER1 under two different control strategies is shown. The red lines denote the control effect under the proposed H_∞ control method with Markov jump. The grey lines denote the control effect under the traditional method without considering Markov jump. In this case, by changing the difference of parameters in different Markov modes, we can show the advantages of the proposed method when the difference of parameters is large. In the four subgraphs of Fig. 9, the parameter deviations of different modes increase from left to right and from top to bottom. Obviously, with the increase of the deviation of parameters of different modes, the control effect of the proposed method is not affected, but the control effect of the traditional method without considering Markov jump is getting worse, even diffuse. Moreover, the voltage stability curve controlled by the proposed method has less fluctuation and is limited to the voltage stability curve controlled by traditional method. Obviously, under the control of the H_∞ strategy proposed in this paper, the DC bus voltage within the edge ER system is more stable and has better performance.

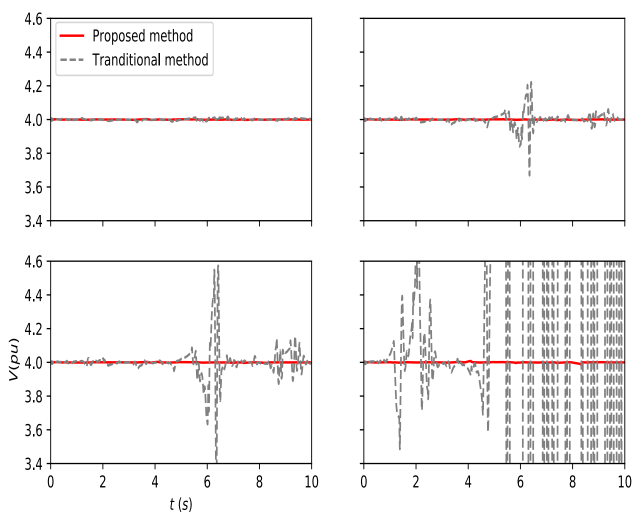


Fig. 9. Comparison between the traditional method and the proposed method

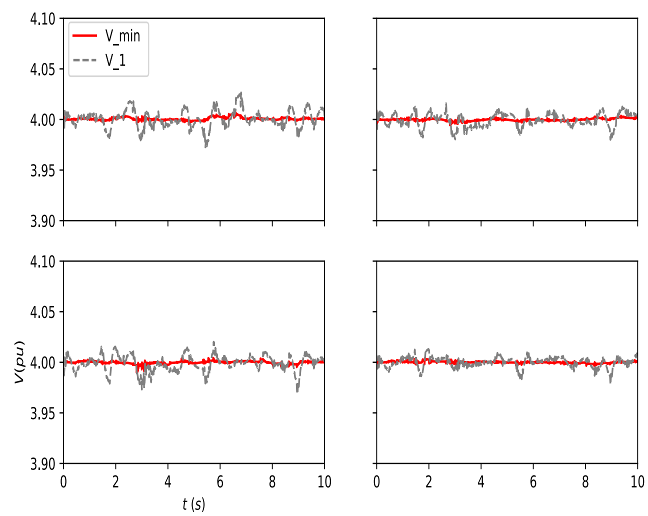


Fig. 10. Comparison between methods in theorem 1 and case 3

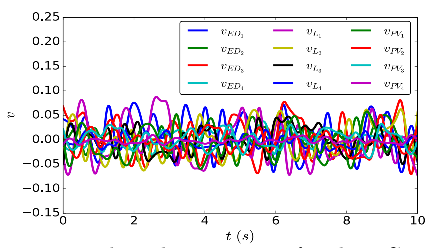
Case 2. In this case, the advantages of the proposed H_∞ control method with Markov jump compared with the traditional H_∞ control method without Markov jump are discussed. The traditional H_∞ control method without considering Markov jump discussed in this paper is to obtain the control signal by solving a single LMI, while the method proposed in this paper actually solves a group of LMI to obtain the control signal suitable for different Markov modes. Because of this characteristic, the proposed method can deal with the change of model

Case 3. In this case, the H_∞ control method proposed in this paper is further optimized to obtain stronger system robustness. For a real physical system, the intensity of external interference may be worse. Therefore, it is very important to enhance the robustness of a controller in the face of higher intensity of external interference. In this case, the external disturbance of the system is increased by 10 times on the basis of case 1 and case 2, and the frequency of Markov jump is doubled. The gray lines in Fig. 10 is the control effect of voltage deviation under the controller based on theorem 1. By comparing the lines in

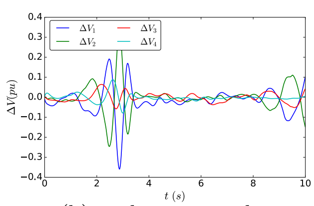
Fig. 5 with the gray line in Fig. 10, the robustness of the controller is not ideal when suppressing higher intensity disturbances. According to formula (17), we can find that when the γ is lower, the robustness of the controller could be better. Therefore, we extend the original theorem 1 to a convex optimization problem. LMI and other inequalities are used as constraints to minimize the square of the γ. The optimization problem is as follows:

$$
\begin{aligned}
& min \quad k\gamma^2 \\
& s.t. \begin{bmatrix} \Omega_i + Ad_i R Ad_i' & X_i' D' & C_i & \Xi_i \\ DX_i & -I & 0 & 0 \\ C_i' & 0 & -\gamma_2 I & 0 \\ \Xi_i' & 0 & 0 & -\Gamma_i \end{bmatrix} \leq 0 \\
& \qquad \gamma >= 0 \\
& \qquad X_i > 0 \\
& \qquad R > 0
\end{aligned}
\tag{28}
$$

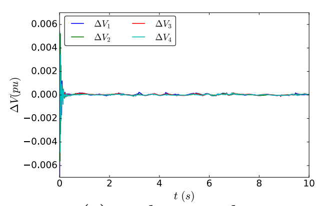
where k is the weight coefficient.



(a) Continuous disturbance inputs for the EC system.



(b) without control



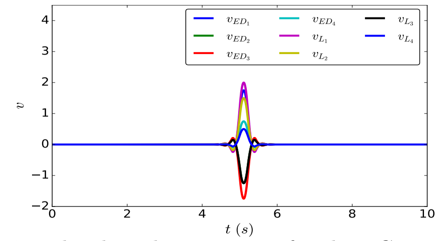
(c) with control

Fig. 11. Frequency deviations under the continuous disturbance

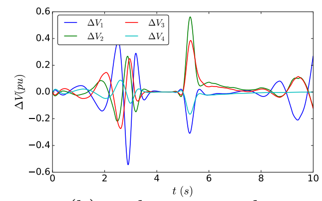
By setting k = 0.01, the minimum value of γ is 0.0193. As shown in Fig. 10, the control effect of DC bus voltage deviation under the controller solved by (28) has the better ability of reducing the negative influence of disturbance on control effect. In this case, by minimizing γ, the H_∞ control method proposed is further optimized. In this paper, the matlab CVX tool is used to solve this optimization problem.

Case 4. The influence of different external disturbances on DC bus voltage deviation is discussed in this case. There are many kinds of external interference to real edge ERs system. Therefore, the effect of the proposed H_∞ control method under two different disturbances is shown in this case.

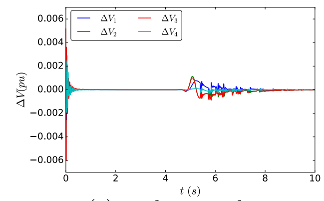
The continuous disturbance is shown in Fig. 11 (a). Comparing with Fig. 11 (b) and Fig. 11 (c), it can be seen that the proposed H_∞ method keeps the voltage deviation



(a) Impulse disturbance inputs for the EC system.



(b) without control



(c) with control

Fig. 12. Frequency deviations under the impulse disturbance

stable under continuous interference. From Fig. 11 (c), it is obvious that the voltage deviation under control fluctuates slightly due to the continuous disturbance.

In order to verify the robustness of the controller under short-term and high-energy disturbance. The impulse disturbance is used as the disturbance input of this edge ER system. By comparing Fig. 12(b) and 12(c), it can be seen that the voltage deviation within the four edge ER nodes is effectively stabilized under the H_∞ proposed control strategy. From Fig. 12 (c), when the impulse disturbance acts on the system at 5 seconds, the voltage deviation has a small vibration, which shows that the proposed control method has a good suppression effect in the face of impulse disturbance. In this case, it shows that the stability of DC bus voltage deviation can be maintained under different external disturbances.

V. Conclusion

In this paper, the DC bus voltage regulation of edge ER system for powering edge task computing is discussed. Considering the dynamics, uncertainty and stochasticity of the system, the external disturbance, and the delay of energy transmission and task scheduling, a morkov jump ODEs with time delay is used to model the system. In order to complete the computing tasks in the edge ER, it is very important to ensure the sufficient and stable energy supply. Considering all the above factors, the DC bus voltage regulation problem is transformed into a morkov jump control problem, and the problem is solved by LMI approach. The effectiveness and feasibility of the proposed method are successfully verified by numerical simulations. And through comparative simulation, the DC bus voltage within the edge ER system controlled by the H_∞ strategy proposed in this paper is more stable and has better performance.

In this paper, we consider the time delay in scheduling tasks and transmitting energy. In the proposed edge ER system, the time delay affects the user experience and even the system security. When the time delay is too large, the control signal may not be transmitted to each device in time, which threatens the DC bus voltage stability. Therefore, how to avoid the control signal packet loss, or in the case of control signal packet loss can still control the DC bus voltage stability is a problem worthy of attention. In the future, we will further consider the control signal loss caused by too high delay, so as to design a voltage stability control method which can control the edge ER well even if some control signals are lost.

Acknowledgment

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