# Probabilistic Power Flow Calculation of Microgrid Based on $\ell_1$-Minimization

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***Abstract*—In view of the impact of the uncertainty of renewable energy on microgrid operation, traditional deterministic power flow calculation becomes more and more difficult to fully describe system operation states and power flow distribution. Considering the randomness and correlation of source and load in a microgrid, this paper establishes a probabilistic power flow model for microgrid systems. The probabilistic power flow solving algorithm we propose is based on $\ell_1$-minimization, which effectively improves the computing efficiency of probabilistic power flow of microgrid with high-dimensional input random variables. By simulating on the modified IEEE 30 node, the accuracy and effectiveness of the proposed method are verified by comparing with the traditional Monte Carlo method.**

***Index Terms*—$\ell_1$-minimization, microgrid, probabilistic power flow (PPF), uncertainty**

## I. Introduction

In order to deal with the global energy crisis and environmental pollution, developing clean energy is a common measure of all nations in the world. A microgrid is a power generation and distribution system that integrates distributed generators (DGs), energy storage systems (ESSs), and loads [1]. It can effectively accommodate clean energy and has a wide application prospect. However, with the increasing penetration of renewable energy, the uncertain factors caused by intermittent energy (such as wind power and solar energy) are increasing, which challenges the stable operation of microgrid. Due to the influence of these uncertainties, traditional deterministic power flow analysis methods are no longer suitable for the analysis of microgrid operation states, thus accounting for the development of probabilistic power flow (PPF) calculation.

PPF calculation is of great significance to ensure the safe and stable operation of microgrid. It takes multiple uncertainties of microgrid into account, analyzes the probabilistic characteristics of power flows, and helps to detect potential system crises. PPF was first proposed by Borkowska in the 1970s [2]. Its essence is to solve the power flow equations with nodal injection power as input random variables and power system state variables as output random variables, so as to obtain the probabilistic characteristics of power systems.

Existing probabilistic power flow solving methods mainly include simulation methods, analytical methods, approximate methods, and polynomial chaos expansion methods:

- Monte Carlo (MC) method and its improved algorithms are widely used as the representative of simulation methods [3], [4]. The basic idea is to sample input variables on a large scale, then get the corresponding sample solutions, and finally analyze the statistical data of the sample solutions. These methods have the advantages of simple principles and convenient operations. However, their convergence rate are usually very low, and large-scale sampling will lead to low calculation efficiency. Large sample MC method is usually used as a comparative standard for the accuracy evaluation of other PPF methods.
- Analytic methods mainly refer to the cumulant method [5]. It has fast calculation speed because it does not need sampling, and the solution of PPF can be acquired through simple arithmetic operations. The disadvantage is nonetheless that the linearization of power flow equations is required. Consequently, when input random variables fluctuate greatly, it is difficult to guarantee the accuracy of the calculation results.
- Point estimate method is a typical approximation method, which uses the probability distribution of known random variables to find the moments of state variables [6]. This method takes less time and has high precision of means and variances of output random variables. However, the accuracy of higher-order moments is too low to provide the probability distribution of output random variables.

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- Polynomial chaos expansion (PCE) is an important method in uncertainty quantification theory [7]. It has been widely used to solve PPF in recent years. In this method, random variables are expanded under a set of multiple standard orthogonal random polynomial basis functions, and then expansion coefficients are obtained by solving equations. The PCE method has good accuracy and efficiency. However, it is not suitable for high-dimensional problems due to the curse of dimensionality.

Based on existing methods, this paper proposes a novel method of applying $\ell_1$-minimization theory from optimization theory in solving microgrid PPF [9], which makes full use of the sparsity of PPF expansion [8]. Although our method draws support from PCE random polynomial expansion in form, it solves expansion coefficients with the sampling method. This avoids solving large-scale complex equations in PCE method and eliminates the problem of dimensionality curse. At the same time, compared with traditional MC method, our method greatly reduces the number of samples and improves the efficiency of MC method.

The rest of this paper is organized as follows: Section II describes the PPF model of the microgrid system; Section III introduces in detail the processing of the correlation of random input variables in PPF and the procedures of $\ell_1$-minimization algorithm; numerical results and relevant analysis are provided in Section IV; Section V finally concludes this paper.

## II. Probabilistic Power Flow Model for Microgrids

The randomness of microgrid power flow considered in this paper mainly comes from the power fluctuation of load nodes and renewable power generation nodes, e.g., photovoltaic (PV) generators and wind power generators. Therefore, this section mainly focuses on the probability models of loads, PV generators, and wind power generators.

### A. Probabilistic Model of Loads

The randomness in the power fluctuation of load nodes results from the influence of environment, time, user behavior, and other uncertain factors. Generally, this randomness can be described by normal distribution with the following probability density function (PDF) [10]:

$$\begin{cases} f(P_L) = \dfrac{1}{\sqrt{2\pi}\sigma_{P_L}} \exp[-\dfrac{(P_L-\mu_{P_L})^2}{2\sigma_{P_L}^2}], \\ \\ f(Q_L) = \dfrac{1}{\sqrt{2\pi}\sigma_{Q_L}} \exp[-\dfrac{(Q_L-\mu_{Q_L})^2}{2\sigma_{Q_L}^2}], \end{cases} \tag{1}$$

where $P_L$ is the load active power, $\mu_{P_L}$ and $\sigma_{P_L}$ are the expectation and standard deviation of $P_L$ respectively, $Q_L$ is the load reactive power, and $\mu_{Q_L}$ and $\sigma_{Q_L}$ are the expectation and standard deviation of $Q_L$ respectively.

### B. Probabilistic Model of Photovoltaic Generators

The randomness of the output power of PV panels mainly comes from the randomness of light intensity. In a fixed period of time, the light intensity approximately follows Beta distribution [11], and the active power of PV generation is directly proportional to the solar radiation intensity. Therefore, the PV active power output also follows Beta distribution with the following PDF:

$$f(\hat{P}_{PV}) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\hat{P}_{PV}\right)^{\alpha-1} \left(1-\hat{P}_{PV}\right)^{\beta-1}, \tag{2}$$

where $\hat{P}_{PV}$ is the ratio of $P_{PV}$, PV active power output, to $P_{PV}^{\max}$, the maximum active power of photovoltaic power supply, $\Gamma(\cdot)$ is Gamma function, and $\alpha$ and $\beta$ are coefficients of Beta distribution. Since the factors of photovoltaic power generators are basically constant, PV reactive power can be determined by PV active power.

### C. Probabilistic Model of Wind Power Generators

Wind power is mainly affected by wind speed, which has randomness according to region, season, temperature, and other factors. The most commonly used probability model to describe wind speed is two-parameter Weibull distribution with the following PDF [12]:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp[-\left(\frac{v}{c}\right)^k], \tag{3}$$

where $v$ is the wind speed, and $k$ and $c$ are the shape parameter and scale parameter of Weibull distribution respectively.

The relationship between wind active power and wind speed satisfies the following piecewise function:

$$P_W(v) = \begin{cases} 0 & v \le v_{ci}, \\ \dfrac{P_r(v-v_{ci})}{v_r - v_{ci}} & v_{ci} \le v \le v_r, \\ P_r, & v_r \le v \le v_{co}, \\ 0 & v \le v_{co}, \end{cases} \tag{4}$$

where $P_w(v)$ is the wind active power at wind speed $v$, $P_r$ is the rated power of wind turbine, and $v_{ci}$, $v_r$, and $v_{co}$ are cut-in, rated, and cut-out wind speeds respectively.

Combining (3) and (4), we can obtain the PDF of the active power of wind power generation:

$$f(P_W) = \begin{cases} 1-\exp[-\left(\frac{v_{ci}}{c}\right)^k] + \exp[-\left(\frac{v_{co}}{c}\right)^k] & P_W = 0, \\ \frac{k}{k_1 c}\left(\frac{P_W - k_2}{k_1 c}\right)^{k-1} \exp[-\left(\frac{P_W-k_2}{k_1 c}\right)^k] & 0 \le P_W \le P_r \\ \exp[-\left(\frac{v_r}{c}\right)^k] - \exp[-\left(\frac{v_{co}}{c}\right)^k] & P_W = P_r, \end{cases} \tag{5}$$

where $k_1$ and $k_2$ are calculated by:

$$k_1 = \frac{P_r}{v_r - v_{ci}}, \quad k_2 = -k_1 v_{ci}.$$

Constant power factor control is often used when wind power generators are connected to the microgrid. Denote the power factor of wind power generation by $\phi_w$. Then the reactive power of wind power generation can be expressed as:

$$Q_W = P_W \tan(\cos^{-1}\phi_W). \tag{6}$$

### D. Probabilistic Power Flow Model

Consider a microgrid system with $M$ nodes in total, and each node can be a load node, PV node, or wind power node. Under given operation conditions, each microgrid node $i$ ($= 1, 2, \dots, M$) needs to maintain power balance at all times, i.e., to meet the following power flow equations [13]:

$$\begin{cases} P_i = V_i \sum_{j=1}^{M} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}), \\ Q_i = V_i \sum_{j=1}^{M} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}), \end{cases} \quad (7)$$

where $P_i$ and $Q_i$ are the active power and reactive power injected into node $i$ respectively, $V_i$ and $V_j$ are the voltage amplitudes of node $i$ and its adjacent node $j$ respectively, $\delta_{ij}$ is the phase difference between $i$ and $j$, and $G_{ij}$ and $B_{ij}$are the real part and the imaginary part of the admittance matrix.

Since the system node injection power includes random loads, PV, and wind power generation, the injected active power $P_i$ and reactive power $Q_i$ are random variables. Therefore, (17) are the PPF equations to be solved. Given the active power and reactive power of a node, the statistical information of voltage magnitude and voltage phase angle of can be calculated, and then the related line power flow can be obtained.

For the convenience of discussion, (17) can be abbreviated as:

$$X = F(Y), \quad (8)$$

where $X$ is the random vector composed of all random variables in the power flow, including the random power of loads, PV, and wind power generation, $Y$ is the vector composed of the state variables to be determined, i.e., the voltage amplitudes and phase angles of all nodes, and $F(\cdot)$ is the function determined by the relationship between random variable $X$ and state variable $Y$ shown by (17).

## III. Solving Probabilistic Power Flow via $\ell_1$-Minimization

The premise of applying $\ell_1$-minimization theory in solving PPF is to calculate the PCE expansion of the random state variables. Based on the sparsity of the expansion coefficients, using $\ell_1$-minimization can subsequently restore state variables. The PCE expansion of random state variables requires that random variables are independent of each other. Unfortunately, in the same microgrid system, random variables usually have correlation. To overcome this problem, we need to preprocess random state variables using Nataf transform before applying PCE expansion. This section first introduces the principle of Nataf transform, and then provides details about the $\ell_1$-minimization algorithm.

### A. Nataf Transform

Nataf transform is used to deal with the correlation between input variables [14]. It is a mathematical method to reconstruct the joint distribution when the marginal distribution of input variables is known.

For any $d$-dimensional input variable $Y = [y_1, y_2, \dots, y_d]$, denote by $[\eta_{Y_{ij}}]_{d\times d}$ its correlation matrix, where

$$\eta_{Y_{ij}} = \frac{cov(y_i, y_j)}{\sigma_i \sigma_j},$$

$\sigma_i$ and $\sigma_j$ are the standard deviation of $y_i$ and $y_j$ respectively, $cov(y_i, y_j)$ is the covariance of $y_i$ and $y_j$, and $\eta_{Y_{ij}}$ is the correlation coefficient of $y_i$ and $y_j$. Then, random variable $Y$ can be transformed into a standard normal distribution variable $Z = [z_1, z_2, \dots, z_d]$ by the following Nataf transform:

$$z_i = \phi^{-1}(\varphi_i(y_i)), \qquad i = 1, 2, ..., d, \quad (9)$$

where $\varphi(\cdot)$ is the cumulative distribution function of $y_i$, and $\phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

In this case, the correlation matrix of $Z$ can be calculated based on $[\eta_{Y_{ij}}]_{d\times d}$, and then $Z$ can be acquired from the standard normal distribution [15]. We can subsequently invert the transform in (9) to obtain the original random variable $X$. As a result, the original PPF equation (8) can be rewritten as follows:

$$X = F(Y) = G(Z). \quad (10)$$

After transforming the random input variable into a standard normal distribution variable, we apply PCE expansion to the random state variable.

### B. Polynomial Chaos Expansion

PCE expansion uses a set of orthogonal polynomials as the basis function, and then intercepts finite terms to approximate random variables [16].

The PPF equation (10) can be expanded by Hermite orthogonal polynomials:

$$X = G(Z) = \sum_{|i|\le n} c_i H_i(Z), \quad (11)$$

where $i = (i_1, i_2, \dots, i_d) \in \mathbb{N}_0^d$ is a multilevel index that satisfies

$$|i| = i_1 + i_2 + \dots + i_d,$$

and $n$ is the order of polynomial expansion. In addition, $H_i(\cdot)$ is the orthogonal Hermite basis function of a $d$-dimensional random variable, which can be calculated as the tensor product of single Hermite basis functions $h_i(\cdot)$:

$$H_i(Z) = h_{i_1}(z_1) h_{i_2}(z_2), ..., h_{i_d}(z_d).$$

Moreover, multiple orthogonal polynomial $H_i(\cdot)$ also satisfies:

$$\mathbf{E}[H_i(Z)H_j(Z)] = \int H_i(Z) H_j(Z) \rho(Z) = \gamma_i \chi_{ij}, \quad (12)$$

where $\chi_{ij}$ is the following Kronecker function:

$$\chi_{ij} = \begin{cases} 1 & i = j, \\ 0 & \text{otherwise}, \end{cases}$$

$\rho(Z)$ is the joint probability density function of random variable $Z$, and $\gamma_i$ is a constant. Note that $c_i$ in (11) are the expansion coefficients to be determined. It has been proved that Hermite orthogonal basis function is the best square approximation of normal random variables satisfying independent distribution. Therefore, the moment and probability distribution of random variables can be determined by calculating the coefficients of the expansion.

Existing PCE expansion methods like Garlerkin projection method and collocation method are prone to the problems of complication in operation and dimensionality disaster [17] [18]. Therefore, in this paper, we use $\ell_1$-minimization method to restore the expansion coefficients based on the sparsity of random variable expansion. In this method, the sample solution is used to restore the coefficients, which not only inherits the simple operation of MC method, but also overcomes the dimensional problem of PCE method.

### C. $\ell_1$ minimization

Note that the expansion coefficient $\mathbf{c}$ in (11) is sparse, $\mathbf{c}$ is a $D \times d$ matrix, which has the form:

$$\mathbf{c} = \begin{pmatrix} c_{11} & c_{21} & \dots & c_{d1} \\ c_{12} & c_{22} & \dots & c_{d2} \\ \dots & \dots & \dots & \dots \\ c_{1D} & c_{2D} & \dots & c_{dD} \end{pmatrix}.$$

According to the sparse reduction theory [19], we can restore $\mathbf{c}$ from the sample solution.

First, we randomly select $N$ sample points $[Z^{(1)}, Z^{(2)}, \dots, Z^{(N)}]$ according to the probability distribution of random variable $Z$. Then, we find $N$ sample solutions $u = [X^{(1)}, X^{(2)}, \dots, X^{(N)}]$ by substituting sample points into PPF equation (10), which also satisfies the following equation:

$$\mathbf{u} = \Psi \mathbf{c}. \tag{13}$$

$\Psi$ is called the measurement matrix, acquired by substituting random sample points $[Z^{(1)}, Z^{(2)}, \dots, Z^{(N)}]$ into multiple Hermite orthogonal polynomials:

$$\Psi = \begin{pmatrix} H_1(Z^{(1)}) & H_2(Z^{(1)}) & \dots & H_D(Z^{(1)}) \\ H_1(Z^{(2)}) & H_2(Z^{(2)}) & \dots & H_D(Z^{(2)}) \\ \dots & \dots & \dots & \dots \\ H_1(Z^{(N)}) & H_2(Z^{(N)}) & \dots & H_D(Z^{(N)}) \end{pmatrix}.$$

$\Psi$ is $N \times D$ dimensional, with $D$ being the number of Hermite basis functions.

Note that when $D > N$, (13) is an underdetermined system and therefore has infinite number of solutions. In order to ensure the uniqueness of the solution, additional constraints are needed. A general way is to reduce the number of expansion basis functions as much as possible, i.e., to increase the sparsity of coefficient $\mathbf{c}$ as much as possible. As a result, adding sparsity constraint to $\mathbf{c}$ yields the following optimization problem:

$$\min \|\mathbf{c}\|_0 \qquad \text{s.t.} \quad \Psi \mathbf{c} = \mathbf{u}, \tag{14}$$

where $\|\cdot\|_0$ represents $\ell_0$ norm, i.e., the number of nonzero elements in $\mathbf{c}$.

However, due to the discontinuity of $\ell_0$ norm, problem (14) is an NP-hard problem, which is difficult to solve. Therefore, we consider the most common way to relax the problem—substituting $\ell_0$ norm with $\ell_1$ norm, the sum of absolute values of all elements in a matrix. Then problem (14) becomes the following $\ell_1$-minimization problem:

$$\min \|\mathbf{c}\|_1 \qquad \text{s.t.} \quad \Psi \mathbf{c} = \mathbf{u}. \tag{15}$$

Existing research shows that the solution of problem (15) can accurately approximate the solution of problem (14). Moreover, solving $\ell_1$-minimization problem is more convenient. In this paper, sampling orthogonal matching pursuit (OMP) algorithm is used to solve the problem [20].

In summary, the whole process of solving PPF is as follows:

**Algorithm 1** Procedure of $\ell_1$-minimization algorithm for PPF

1. Perform Nataf transform (10) based on probability distribution of random input $X$;
2. Apply PCE expansion (11) to the transformed PPF equation;
3. Randomly select $N$ sample points $[Z^{(1)}, Z^{(2)}, \dots, Z^{(N)}]$ from normal distribution, and obtain sample solutions $\mathbf{u} = [X^{(1)}, X^{(2)}, \dots, X^{(N)}]$ by Newton-Raphson method [21];
4. Construct $\ell_1$-minimization problem (15);
5. Obtain coefficient $\mathbf{c}$ by sampling OMP algorithm;
6. Substitute $\mathbf{c}$ back into expansion (11), obtain the statistical information of state variable $Y$, and carry out further analysis.

## IV. Numerical Experiment

We verify Algorithm 1, our PPF solving algorithm, on standard IEEE 30 node. The accuracy and efficiency of the algorithm are evaluated by comparing with MC method. The PPF calculation program is developed by Matlab R2020, and the power flow calculating part is implemented by Matpower software package. The program is run on a platform with Intel i7-8586u CPU and 2G memory independent graphics card.

### A. Parameter Setting

In this paper, based on IEEE 30 node grid structure, two wind turbines are connected to nodes 7 and 25, and two photovoltaics are connected to nodes 2 and 19. Node 2 is set as PV node, and other wind turbines and photovoltaic nodes are treated as PQ nodes. At the same time, nodes 3, 12, 18, and 26 are set as load nodes with random fluctuations. System line parameters and reference capacity are consistent with the 30 node example of Matpower.

The parameters of wind turbines, photovoltaic, and random loads are given in Table I, II, and III respectively.

The direct correlation coefficient of two wind turbines is 0.6, the correlation coefficient of photovoltaic power is 0.8, and

TABLE I
PARAMETER SETTING FOR WIND TURBINES

| **WT** | $P_r$ (MW) | $v_{ci}$ | $v_r$ | $v_{co}$ | $c$ | $k$ |
|---|---|---|---|---|---|---|
| WT1 | 15 | 3 | 20 | 10.7 | 4 | 0.9 |
| WT2 | 14 | 3 | 22 | 8 | 3 | 0.9 |

TABLE II
PARAMETER SETTING FOR PHOTOVOLTAICS

| **PV** | $P_{PV}^{max}$ (MW) | $\alpha$ | $\beta$ | $\phi_{PV}$ |
|---|---|---|---|---|
| PV1 | 50 | 0.9 | 0.85 | 0.95 |
| PV2 | 80 | 0.8 | 0.75 | 0.95 |

the correlation coefficient matrix of four random loads is as follows:

$$\rho_{load} = \begin{pmatrix} 1 & 0.6 & 0.2 & 0.7 \\ 0.6 & 1 & 0.3 & 0.4 \\ 0.2 & 0.3 & 1 & 0.4 \\ 0.7 & 0.4 & 0.8 & 1 \end{pmatrix}. \tag{16}$$

*B. Simulation results*

In order to verify the accuracy of the $\ell_1$-minimization-based PPF method, the calculation results of simple random sampling MC method with 500,000 sampling times are taken as reference. We analyze the simulation results from the accuracy and efficiency of the algorithm.

*1) Accuracy:* In this paper, the accuracy of the algorithm is described by the average relative errors between the expectations and standard deviations of the output voltage amplitude and phase angle of each node:

$$\begin{cases} \bar{\mu} = \sum_{i=1}^{M} \left( \frac{\mu_{\ell_1} - \mu_{MC}}{\mu_{MC}} \right) / M, \\ \bar{\sigma} = \sum_{i=1}^{M} \left( \frac{\sigma_{\ell_1} - \sigma_{MC}}{\sigma_{MC}} \right) / M. \end{cases} \tag{17}$$

Table IV shows the average and standard deviation of voltage amplitude $V$ and phase angle $\delta$ calculated by Algorithm 1 and the error value of MC method of 500000 times as reference.

TABLE III
PARAMETER SETTING FOR LOADS

| **Load** | $\mu_{P_L}$ (MW) | $\mu_{P_Q}$ (MW) | $\sigma_{P_L}$ (MW) | $\sigma_{Q_L}$ (MW) |
|---|---|---|---|---|
| Load1 | 2.4 | 1.2 | 0.12 | 0.06 |
| Load2 | 11.2 | 7.5 | 0.56 | 0.375 |
| Load3 | 3.2 | 0.9 | 0.16 | 0.045 |
| Load4 | 3.5 | 2.3 | 0.175 | 0.115 |

TABLE IV
ERROR ESTIMATION

| $\ell_1$-Minimization | $\bar{\mu}_V$ | $\bar{\sigma}_V$ | $\bar{\mu}_\delta$ | $\bar{\sigma}_\delta$ |
|---|---|---|---|---|
| Max Error | 1.59e−6 | 5.08e−7 | 8.14e−5 | 6.75e−4 |
| Average Error | 2.33e−7 | 1.71e−7 | 3.53e−5 | 1.91e−45 |

Table IV shows that the expectation and standard deviation of output state variables of probabilistic power flow obtained by $\ell_1$-minimization have high accuracy. Take node 12 as an example. The PDFs of voltage amplitude and phase angle of node 12 obtained by $\ell_1$-minimization algorithm are given in Fig. 1. It can be seen that the distribution function obtained by $\ell_1$-minimization basically coincides with that obtained by MC method, which further illustrates the accuracy of our algorithm.

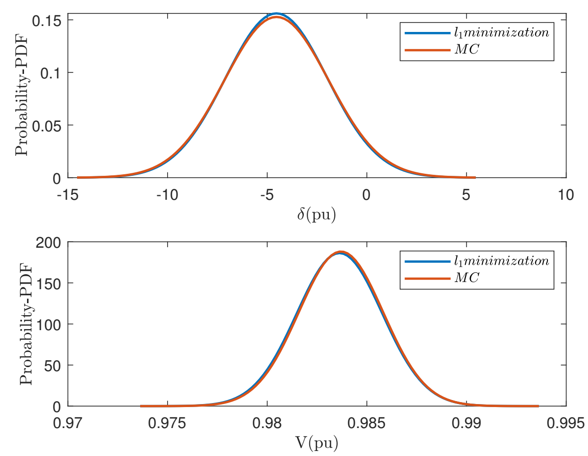


Fig. 1. Probability density function for $V$ and $\delta$ of node 12.

Furthermore, Fig. 2 shows the sparsity of PCE expansion coefficients of voltage amplitude and phase angle of 30 nodes obtained by Algorithm 1. In this example, the highest order of the basis function is 3, and the dimension of the random variable is 16, so the number of basis functions is

$$P = \frac{(16+3)!}{16!3!} = 969.$$

That is, the dimension of expansion coefficient $\mathbf{c}$ is 969. The sparsity $s$ in the figure is calculated by:

$$s = \frac{\#\{|\mathbf{c}| \geq \tau\}}{969}, \tag{18}$$

where $\#\{|\mathbf{c}| \geq \tau|\}$ represents the number of elements in coefficient matrix c that is greater than or equal to threshold $\tau$.

It can be seen from Fig. 2 that the sparsity of voltage amplitude and phase angle of each node under the expansion of Hermite orthogonal basis is less than 3%, which proves the applicability of $\ell_1$-minimization in solving PPF.

*2) Efficiency:* We continue to analyze the efficiency advantages of $\ell_1$-minimization algorithm.

Fig. 3 shows the convergence comparison between the errors of expected voltage amplitudes obtained by $\ell_1$-minimization algorithm and MC method. It can be seen from the figure that the errors of the two methods decrease as the number of samples increases. Then the error of $\ell_1$-minization algorithm converges rapidly with the number of samples. Moreover, when the number of samples reaches a certain level, the error

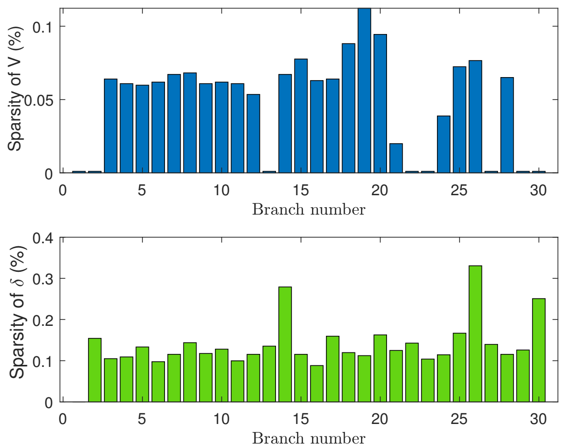


Fig. 2. Sparsity of coefficient **c**.

is basically unchanged if the number of samples continues to increase. In contrast, error convergence of MC method is slower. Under the same sample level, the error of $\ell_1$-minimization algorithm is obviously stronger than that of MC method. The error analysis results of other output state variables are similar and will not be further explained.

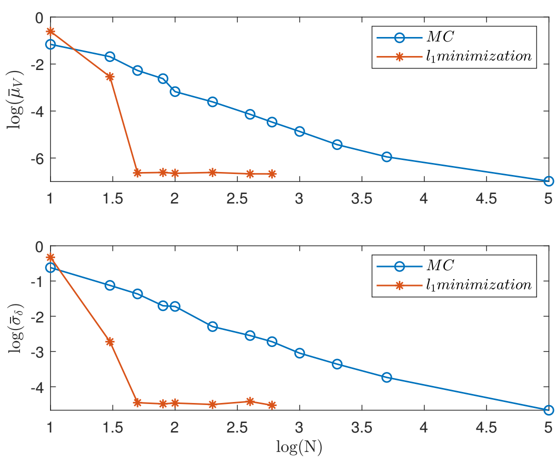


Fig. 3. Error estimates for $\ell_1$ minimization and MC method.

Table V shows the sample sizes and time consumption of $\ell_1$-minimization algorithm and MC algorithm under the same error level. It can be seen from the statistics that in order to achieve the same accuracy, the operation time of $\ell_1$-minimization algorithm is about 1/20 of MC method, which greatly improves the operation efficiency of PPF. In addition, the sample solution is used to solve the problem just like MC method, which simplifies the implementation process of the algorithm.

*3) Summary:* Based on the above statistics and figures, the applicability and accuracy of $\ell_1$-minimization algorithm in solving PPF problems are fully illustrated. Moreover, $\ell_1$-minimization can greatly improve the efficiency on the premise of ensuring accuracy. Consequently, it can be widely used to solve PPF instead of MC method.

TABLE V
COMPARISON OF CALCULATION EFFICIENCY

| | $\bar{\mu}_V$ | $\bar{\sigma}_V$ | samples | time cost (s) |
|---|---|---|---|---|
| $\ell_1$-minimization | 2.33e−7 | 5.53e−5 | 50 | 12.187 |
| MC | 1.02e−7 | 2.155e−5 | 10000 | 1992.643 |

## V. CONCLUSION

In this paper, the input uncertainty of the microgrid system is considered. First, the PPF model of a microgrid is established. Considering that the nodes in the same microgrid have correlation, Nataf transform is used to decorrelate the variables. Then, a PPF computing method based on $\ell_1$-minimization theory is proposed to solve the corresponding output random variables of microgrid system. In this method, given the sparsity of PPF random state variable expansion, with the number of sample solutions much smaller than that required by MC method, we can accurately restore the corresponding random state variables of PPF in microgrid. Numerical results show that the $\ell_1$-minimization-based method can greatly improve the efficiency of microgrid power flow computing. Compared with PCE and other methods, it has simpler principles and is easier to implement. Based on the research of this paper, our future research will continue to improve $\ell_1$-minimization algorithm. We will consider using reweighted $\ell_1$-minimization algorithm to further improve the efficiency of solving.

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